Flux saturation length of sediment transport

Thomas Pähtz\textsuperscript{1,2}, Jasper F. Kok\textsuperscript{3}, Eric J. R. Parteli\textsuperscript{4} and Hans J. Herrmann\textsuperscript{5,6}

1. Department of Ocean Science and Engineering, Zhejiang University, 310058 Hangzhou, China.
2. State Key Laboratory of Satellite Ocean Environment Dynamics, Second Institute of Oceanography, Hangzhou, China.
3. Department of Earth and Atmospheric Sciences, Cornell University, Ithaca, NY 14850, USA.
4. Institute for Multiscale Simulation, Universität Erlangen-Nürnberg, Nägelsbachstraße 49b, 91052 Erlangen, Germany.
5. Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil.
6. Computational Physics, IFB, ETH Zürich, Schafmattstrasse 6, 8093 Zürich, Switzerland.

Sediment transport along the surface drives geophysical phenomena as diverse as wind erosion and dune formation. The main length-scale controlling the dynamics of sediment erosion and deposition is the saturation length $L_s$, which characterizes the flux response to a change in transport conditions. Here we derive, for the first time, an expression predicting $L_s$ as a function of the average sediment velocity under different physical environments. Our expression accounts for both the characteristics of sediment entrainment and the saturation of particle and fluid velocities, and has only two physical parameters which can be estimated directly from independent experiments. We show that our expression is consistent with measurements of $L_s$ in both aeolian and subaqueous transport regimes over at least five orders of magnitude in the ratio of fluid and particle density, including on Mars.

PACS numbers: 45.70.-n, 47.55.Kf, 92.40.Gc

where $M$ is the mass of sediment in flow per unit soil area, and $V$ is the average particle velocity. Since the fluid loses momentum to accelerate the particles, $Q$ is limited by a steady-state value, the saturated flux $Q_s$, which is largely set by the fluid density $\rho_f$ and the fluid shear velocity, $u_s$ [1–4, 6]. In typical situations, such as on the streamward side of dunes, the deviation of $Q$ from $Q_s$ is small, that is, $|1 - Q/Q_s| \ll 1$ [10, 11, 16]. The rate $\Gamma(Q)$ of the relaxation of $Q$ towards $Q_s$ in the downstream direction ($x$) can thus be approximately written as [7, 10, 16],

$$\Gamma(Q) = \frac{dQ}{dx} \equiv \frac{Q_s - Q}{L_s},$$

where $\Gamma$ is Taylor-expanded to first order around $Q = Q_s$ (\(\Gamma(Q_s) = 0\)), and the negative inverse Taylor-coefficient gives the saturation length, $L_s$. Flux saturation is controlled by the downstream evolution of $M$ and $V$ towards their respective steady-state values, $M_s$ and $V_s$. Changes in $M$ with $x$ are controlled by particle entrainment from the sediment bed into the transport layer. In the aeolian regime (dilute fluid such as air), entrainment occurs predominantly through particle impacts [6], whereas in the subaqueous regime (dense fluid such as water) entrainment occurs mainly through fluid lifting [2, 3]. On the other hand, the evolution of $V$ towards $V_s$ is mainly controlled by the acceleration of the particles due to fluid drag, and their deceleration due to grain-bed collisions [10, 12]. For simplicity, previous studies neglected either the saturation of $V$ [10, 17] or the relaxation of $M$, as well as changes in $V$ due to grain-bed collisions [7, 12]. Moreover, all previous studies [7, 10, 12, 17] did not account for the relaxation of the fluid velocity ($U$) towards its steady-state value ($U_s$) within the transport layer. This relaxation is driven by changes in the transport-flow feedback resulting from the relaxation of $M$ and $V$.

In this Letter, we derive a theoretical expression for $L_s$...
which encodes all aforementioned relaxation mechanisms and is consistent with measurements over at least 5 orders of magnitude in the ratio of the fluid and particle density, without requiring fitting to any of these measurements. Indeed, since previously proposed relations for $L_s$ neglect some of the interactions that determine $L_s$ [7, 10–12], it is uncertain how to adapt these equations to compute $L_s$ in extraterrrestrial environments, such as Mars [5, 6, 13]. Our theoretical expression overcomes this problem, since it is valid for arbitrary physical environments for which turbulent fluctuations of the fluid velocity, and thus transport as suspended load [6], can be neglected. For aeolian transport under terrestrial conditions, this regime corresponds to $u_* \leq 4u_h$, where $u_h$ is the threshold $u_*$ for sustained transport [2, 3, 6].

The first step towards obtaining an expression for $L_s$ is to obtain the momentum conservation equation describing the momentum change of the transport layer. We start from the momentum conservation equation for steady ($\partial \rho / \partial t = 0$) dilute granular flows [18],

$$\frac{\partial (\rho v_x^2)}{\partial x} + \frac{\partial (\rho (v_x v_z))}{\partial z} = (f_x),$$

where $\langle \rangle$ denotes the ensemble average, $\rho$ the mass density, $v$ the particle velocity, and $f$ the external body force per unit volume applied on a sediment particle. Here $f$ incorporates the main external forces acting on the transported particles: drag, gravity, buoyancy, and added mass. The added mass force arises because the speed of the fluid layer immediately surrounding the particle is closely coupled to that of the particle, thereby enhancing the particle’s inertia by a factor $c_a = 1 + 0.5s^{-1}$, where $s = \rho_p / \rho_l$ is the grain-fluid density ratio [2]. Although this added mass effect is negligible in aeolian transport ($c_a \approx 1$), it affects the motion of particles in the subaerial regime [2]. Integration of Eq. (2) over the entire transport layer depth ($\int_0^\infty \cdot dz$) yields,

$$- \frac{d(c_v M V_s^2)}{d x} = \frac{3M}{4c_a s d} \cdot C_d(V_s) \cdot V_r^2 + \langle (v_x v_z) \rangle(0),$$

where $M = \int_0^\infty \rho dz$, $V_s = \int_0^\infty \rho (v_x) dz / M$, and $c_v = \int_0^\infty \rho (v_x^2) / M V_s^2 dz$. In Eq. (3), the quantity $\langle (v_x v_z) \rangle(0)$ gives the difference between the average horizontal momentum of particles impacting onto $\langle (\rho (v_x v_z))_0 \rangle(0)$ and leaving $\langle (\rho (v_x v_z)_1(0) \rangle(0)$ the sediment bed per unit time and soil area. This momentum change is consequence of the collisions between particles within the sediment bed ($z \leq 0$). Thus, $\langle (v_x v_z) \rangle(0)$ is an effective frictional force which the soil applies on the transport layer per unit soil area. It is proportional to the normal component of the force which the transport layer exerts onto the sediment bed [3, 10, 19], $\langle (v_x v_z) \rangle(0) = -\mu g M / c_a$, where $\mu$ is the associated Coulomb friction coefficient, and $g = g(s-1)/s$ is the buoyancy-reduced gravitational constant $g$. In order to obtain the momentum conservation equation of the particles within the transport layer from Eq. (3), we first note that $\int_0^\infty (f_x^{\text{drag}}) dz \approx \frac{3M}{4c_a s d} \cdot C_d(V_s) \cdot V_r^2$ [19], where $d$ is the mean grain diameter and $V_s = U - V$ is the difference between the average fluid ($U$) and particle velocity ($V$). By further noting that the change of $c_v$ with $x$ is negligible (see Suppl. Mat. [20]), we obtain,

$$c_v \frac{d(M V_s^2)}{d x} = \frac{3M}{4c_a s d} \cdot C_d(V_s) \cdot V_r^2 - \mu / c_a \bar{g} M.$$  

Next, we solve Eq. (4) for $\frac{dV_s}{dV}$ thus obtaining an equation of the form $\frac{dV_s}{dV} = \Omega(V)$, and we expand $\Omega(V)$ around saturation, that is, $\Omega(V) \approx \Omega(V_s) + (V-V_s) \Omega'(V)$, where $\Omega(V)$ and $\Omega(V_s) = 0$, we obtain $L_s = -(d\Omega/dQ)^{-1}|_{Q=Q_s} = -(d\Omega/dV)^{-1}|_{V=V_s}$, which leads to

$$L_s = c_v c_a (1 + c_M) V_s V_r F K \cdot [\mu g]^{-1},$$

where $c_M = \frac{V_s}{V_r} \frac{dM}{dV}(V_s)$, and $K = (1 - \frac{\nu}{dL})^{-1}$, while $V_s$ (the steady-state value of $V_s$) and $F$ are given by,

$$V_s = \left[ \sqrt{8 \mu s g d / (9 + (8s / d)^2) - 8 \nu / d} \right],$$

and,

$$F = [V_s + 16\nu / d] \cdot [2V_s + 16\nu / d]^{-1},$$

respectively, where $\nu$ is the kinematic viscosity. Eqs. (6) and (7) result from using $C_d(V_s) = \frac{4\nu}{dL_c} + 1.5$ (valid for natural sediment [21]). We find that using other reported drag laws only marginally affects the value of $L_s$. Furthermore, we note that in the subaqueous regime $c_M \approx 0$, since in this regime $M$ changes within a time-scale which is more than one order of magnitude larger than the timescale over which $Q$ changes [22]. This difference in timescales implies $\nu dM / dQ$ and thus $\nu dM / dV$ in the subaqueous regime. In contrast, in the aeolian regime, $c_M \approx 1$ as the total mass of ejected grains upon grain-bed collisions is approximately proportional to the speed of impacting grains [23], which yields $M / M_s \approx V_s / V_r$. In Eq. (5), the quantity $K$ encodes the effect of the relaxation of the transport-flow feedback, neglected in previous works [7, 10, 17]. In the subaqueous regime, this transport-flow feedback has a negligible influence on the fluid speed [22] (and thus on its relaxation). In this regime, $\frac{dV}{dV} (V_s) \approx 0$, which yields $K \approx 1$ and thus,

$$L_s^{\text{subaq}} = [2c_v + c_v / s] V_s V_r F K \cdot [\mu g]^{-1}.$$  

In contrast, in the aeolian regime, $U$ scales with the shear velocity at the bed ($u_h$) [19, 22], and thus $\frac{dV}{dV} (V_s) \approx \frac{u_h}{\rho_l g}$. Using the mixing length approximation of inner turbulent boundary layer equations [24], $u_h$ can be expressed as $u_h = u_* [1 - 3M C_d(V_s) / (4s \rho_l d_c \rho_l u_s^2)]^{1/2}$ [22]. By using this expression to compute $\frac{dV_s}{dV}$ and noting that $u_{hs} \approx u_*$ [6], we obtain the following expression for $K$,

$$K = \frac{1 + F^{-1} [(V_s + V_r) / (2V_r)] \cdot ([u_* / u_s]^2 - 1]}{1 + [(V_s + V_r) / (2V_r)] \cdot ([u_* / u_s]^2 - 1)}.$$
Using Eq. (9) to compute $K$, the saturation length in the aeolian regime of transport is then given by,
\begin{equation}
L_{s}^{\text{aeolian}} = 3c_vV_sF_k \cdot [\mu g]^{-1}.
\end{equation}

We note that $K$ can be approximated as its limit for large dimensionless shear velocities ($(u_s/u_t)^2 \gg 1$), giving $K \approx V_s/(F_k)$. This approximation has an error typically smaller than 30% for Earth and Mars conditions with $d \geq 200\mu m$ [20]. The benefit of this approximation is that it results in a simple expression for $L_s$ for aeolian transport, $L_{s}^{\text{aeolian}} \approx 3c_vV_s^2 \cdot [\mu g]^{-1}$.

Therefore, from our general expression for $L_s$ (Eq. (5)) we obtain two expressions — Eqs. (8) and (10) — which can be used to predict $L_s$ in the subaqueous and aeolian transport regimes, respectively. Both Eqs. (8) and (10) are consistent with the behavior of $L_s$ with $u_s$ observed in experiments. Indeed, $L_s$ mainly depends on $u_s$ via the average particle velocity, $V_s$. For subaqueous transport, in which $V_s$ is a linear function of $u_s$, $L_s$ varies linearly with $V_s$ and thus with $u_s$, which is consistent with experiments [8]. In contrast, $V_s$ depends only weakly on $u_s$ for aeolian transport [6, 22]. Consequently, $L_s$ is only weakly dependent on $u_s$ in this regime, which is also consistent with experiments [7]. In fact, when neglecting this weak dependence on $u_s$, Eq. (10) reduces to $L_s \propto s/d$ [7, 12] in the limit of large particle Reynolds numbers $\sqrt{sgd^{3}/\nu}$ for which $V_s \propto \sqrt{sgd}$ [22]. Moreover, we estimate the average particle velocity $V_s$ as a function of $u_s/u_t$ using well-established theoretical expressions which were validated against experiments of sediment transport in the aeolian or in the subaqueous regime. Specifically, we use the model of Ref. [19] for obtaining $V_s(u_s/u_t)$ in the aeolian regime and the model of Ref. [25] for the subaqueous regime (see Suppl. Mat. [20]).

The squares in Fig. 1 denote wind tunnel measurements of $L_s$ for different values of $u_s$. These data were obtained by fitting Eq. (1) to the downstream evolution of the sediment flux, $Q(x)$, close to equilibrium [7]. Further estimates of $L_s$ for aeolian transport under terrestrial conditions have been obtained from the wavelength ($\lambda$) of elementary dunes on top of large barchans [7, 20]. These estimates correspond to the circles in Fig. 1, whereas the coloured lines in this figure denote $L_s/(sd)$ versus $u_s/u_t$ predicted by Eq. (10). As we can see in Fig. 1, in spite of the scatter in the data, Eq. (10) yields reasonable agreement with the experimental data without requiring any fitting to these data. In contrast, the scaling $L_s \propto 2sd$ was obtained from a fit to the data displayed in Fig. 1. Moreover, Fig. 2 shows values of $L_s$ estimated from experiments on subaqueous transport under different shear velocities (symbols). These estimates were obtained from measurements of $\lambda$ [12, 27] and from the minimal cross-stream width, $W \approx 12L_s$ [13], of barchans in a water flume [8]. The coloured lines show the behavior of $L_s$ with $u_s$ as predicted from Eq. (8) for subaqueous sand transport [8, 12, 27]. We note that Eq. (8) is the first expression for $L_s$ that shows good agreement with measurements of $L_s$ under water. Indeed, the scaling relation $L_s = 2sd$ [7, 12], which was developed for the aeolian regime, does not capture the increasing trend of $L_s$ with $u_s/u_t$ evident from the experimental data.

An excellent laboratory for further testing our model is the surface of Mars, where the ratio of grain to fluid density ($s$) is about two orders of magnitude larger than on Earth. We estimate the Martian $L_s$ from reported values of the minimal crosswind width $W$ of barchans at

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Dimensionless saturation length, $L_s/(sd)$, versus dimensionless shear velocity, $u_s/u_t$, for aeolian transport under terrestrial conditions. Brown squares denote estimates of $L_s$ from wind-tunnel measurements ($d = 120\mu m$), while the error bars are due to uncertainties in the measurements of the sediment flux [7]. Green circles denote $L_s$ obtained from the wavelength of elementary dunes on top of large barchans ($d = 185\mu m$), whereas the associated error bars contain uncertainties in the dune size [7] (potential systematic uncertainties [20] are not included). The coloured lines represent predicted values of $L_s$ using Eq. (5) for the corresponding experimental conditions ($\rho_p = 2650\, \text{kg/m}^3$, $\rho_t = 1.174\, \text{kg/m}^3$ and $\nu = 1.59 \times 10^{-5}\, \text{m}^2/\text{s}$). The horizontal orange line indicates the prediction of $L_s$ using the scaling $L_s = 2sd$ [7, 12]. The upper legend displays the values of the coefficient of determination ($R^2$) for both expressions.}
\end{figure}
two dune fields on Mars: one at Arkhangelsky crater in the southern highlands and the other near the north pole [13, 20]. However, using Eq. (10) to predict \( L_s \) on Mars is difficult because both the grain size \( d \) and the typical shear velocity \( u_{\text{typ}} \) for which the dunes were formed are poorly known. Indeed, we need to know both quantities to calculate \( V_s \) [19]. We thus predict the Martian \( L_s \) using a range of plausible values of \( d \) and \( u_{\text{typ}} \). Specifically, we assume \( d \) to lie in the broad range of 100 – 600 \( \mu \)m based on recent studies [5]. Estimating \( u_{\text{typ}} \) on Mars is also difficult, both because of the scarcity of wind speed measurements [28], and because the threshold \( u_t \) required to initiate transport (\( u_{\text{typ}} \)) likely exceeds \( u_t \) by up to a factor of \( \sim 10 \) [6, 19, 29]. We therefore calculate \( L_s \) for two separate estimates of \( u_{\text{typ}} \): the first using \( u_{\text{typ}} = u_t \), consistent with previous studies [13, 30], and the second calculating \( u_{\text{typ}} \) based on the wind speed probability distribution measured at the Viking 2 landing site [20], which results in an estimate of \( u_{\text{typ}} \) closer to \( u_t \). Fig. 3 shows that the values of \( L_s \) predicted with either of these estimates are consistent with those estimated from the minimal barchan width. This good agreement suggests that the previously noted overestimation of the minimal size of Martian dunes [31] is largely resolved by accounting for the low Martian value of \( u_t/ut \) [19] and the proportionally lower value of the particle speed \( V_s \), as hypothesized in Ref. [29]. Indeed, the scaling \( L_s = 2sd \) [7, 12] (inset of Fig. 3) requires \( d \sim 29\mu \)m and \( d \sim 40\mu \)m to be consistent with \( L_s \) for the north polar and Arkhangelsky dune fields, respectively. However, such particles, which are smaller than the fines within granule ripples on Mars [30] (87 ± 15\( \mu \)m), are most likely transported as suspended load in the Martian atmosphere [30], as they are on Earth [4].

Finally, Fig. 3 also compares Eq. (8) to measurements of \( L_s \) for Venusian transport, which have been estimated from the wavelength of elementary dunes produced in a wind-tunnel mimicking the Venusian atmosphere [32].

In conclusion, our expression for the saturation length \( L_s \) of sediment flux (Eq. (5)) is the first capable of quantitatively reproducing measurements of \( L_s \) under different flow conditions in both air and under water, and is in agreement with measurements over at least 5 orders of magnitude of variation in the sediment to fluid density ratio. The capability of our expression to reproduce mea-
measurements of \( L_s \) in a wide range of physical environments without requiring fitting to any of the experimental data attests for the significant advance in the understanding of the physics of sediment transport achieved through our model for the saturation length. The future application of our expression thus has the potential to provide important contributions to calculate sediment transport, the response of saltation-driven wind erosion and dust aerosol emission to turbulent wind fluctuations, and the dynamics of sediment-composed landscapes under water, on Earth’s surface and on other planetary bodies.

We acknowledge support from grants NSFC 41350110226, ETH-10-09-2, NSF AGS 1137716, and DFG through the Cluster of Excellence “Engineering of Advanced Materials”. We thank Miller Mendoza and Robert Sullivan for discussions, and Jeffery Hollingsworth for providing us with the pressure and temperature at the Martian dune fields.

[20] See Supplemental Material for a description of how we estimated the model parameters \( c_v \) and \( \mu \), the average sediment velocity \( V_s \), and the saturation length of sediment transport from the size of dunes under water and on planetary bodies.
Flux saturation length of sediment transport — Supplemental Material —

Thomas Pähtz¹,², Jasper F. Kok³, Eric J. R. Parteli⁴ and Hans J. Herrmann⁵,⁶

¹. Department of Ocean Science and Engineering, Zhejiang University, 310058 Hangzhou, China.
². State Key Laboratory of Satellite Ocean Environment Dynamics, Second Institute of Oceanography, Hangzhou, China.
³. Department of Earth and Atmospheric Sciences, Cornell University, Ithaca, NY 14850, USA.
⁴. Institute for Multiscale Simulation, Universität Erlangen-Nürnberg, Nügelsbachstraße 49b, 91052 Erlangen, Germany.
⁵. Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil.
⁶. Computational Physics, IfB, ETH Zürich, Schafmattstraße 6, 8093 Zürich, Switzerland.

This Supplemental Material is organized as follows. In Section I we derive the values of the steady-state particle speed square correlation $c_v$ and the Coulomb friction coefficient $\mu$ — the only parameters of our theoretical expression of the saturation length — from independent experiments of sediment transport in air and under water. Moreover, in Section II we show the analytical expressions used to calculate the steady-state average particle velocity $V_s$ in the aeolian regime and in the subaqueous regime of transport. Next, in Section III, we discuss our estimation of the saturation length from the scale of dunes in different environments. We then present the values of $L_s$ estimated from the size of dunes on Earth, on Mars, on Venus and under water, which we use to compare with the predictions from our theoretical expression for $L_s$ in Fig. 3 of the paper. Finally, in Section IV, we compare the predictions for $L_s$ on Earth and Mars obtained from our theoretical expression for $L_s$ in the aeolian regime (Eq. (10) of the paper) with the predictions obtained using a simplified version of Eq. (10), which is valid for large values of $u_s/u_t$.

I. ESTIMATING $c_v$ AND $\mu$ FROM EXPERIMENTAL DATA

Our theoretical expression for $L_s$ has two empirical quantities which must be estimated from experimental measurements. These empirical quantities are the Coulomb friction coefficient ($\mu$) and the particle speed square correlation ($c_v$). In this Section, we describe how we estimate these quantities from existing experimental data of sediment flux in both aeolian and subaqueous transport regimes.

A. The particle speed square correlation, $c_v$

The quantity $c_v$, that is, the particle speed-square correlation, is defined by the equation,

$$c_v = \frac{\int_0^\infty \rho(v_x^2)dz}{MV^2} = \frac{M \int_0^\infty \rho(v_x^2)dz}{\left( \int_0^\infty \rho(v_x)dz \right)^2},$$

where $M = \int_0^\infty \rho dz$, $V = \int_0^\infty \rho(v_x)dz/M$, $\rho$ is the particle mass density, $v$ is the particle velocity, and $\langle \rangle$ denotes ensemble averaging. That is, the normalized variance of the velocity distribution of the transported particles is then written as $\delta_v = c_v - 1$, where $c_v$ is given by Eq. (1).

First, we note that for transport in the equilibrium ($\partial/\partial x = 0$), $\left( \int_0^\infty \rho(v_x)dz \right)^2 / \int_0^\infty \rho(v_x^2)dz$ is proportional to $u_t^2 - u_s^2$ for both transport regimes, where $u_t$ is the transport threshold [1]. Hence, since most of the transport occurs above the sediment bed ($z > 0$), also the quantity $\left( \int_0^\infty \rho(v_x^2)dz \right)^2 / \int_0^\infty \rho(v_x^2)dz$ is nearly proportional to $u_t^2 - u_s^2$ in the equilibrium. On the other hand, it is known from experiments that $M$ approximately scales with $u_t^2 - u_s^2$ [2,4]. Therefore, due to Eq. (1), $c_v$ is nearly independent of $u_t$ for equilibrium transport. By considering that $c_v$ is nearly independent of $u_t$ for equilibrium transport, as discussed above, it seems reasonable that changes of $c_v$ with $x$ during the saturation process of the sediment flux close to the equilibrium can be regarded as negligible compared to the corresponding changes of $M$ or $V$ with $x$. In this manner, we disregard changes of $c_v$ with $x$ in the derivation of the saturation length equation, as mentioned in the main document (cf. text before Eq. (4)). In other words, the value of $c_v$ used in the saturation length equation corresponds to the steady-state value of the particle speed square correlation. In the following we show how we estimate $c_v$ for transport in the aeolian and subaqueous regimes. For this purpose, it is helpful to rewrite $c_v$ as,

$$c_v = \frac{\langle v_x^2 \rangle}{\langle v_x \rangle^2},$$

where the overbar denotes the height average of a quantity according to the expression,

$$\overline{A} = \frac{\int_0^\infty A \rho dz}{\int_0^\infty \rho dz} = \frac{0}{M}.$$

1. Aeolian regime

It was shown from experiments of sediment transport in a wind tunnel [3] that the particle concentration pro-
file $\rho(z)$ and the particle velocity profile $\langle v_z \rangle(z)$ behave according to the following equations,

$$\rho(z) = \rho(h_o)e^{-(z-h_o)/z_p}, \quad (4)$$

$$\langle v_z \rangle(z) = \langle v_z \rangle(h_o) + m(z-h_o), \quad (5)$$

where $z_p \approx 10 \text{mm}$, $\langle v_z \rangle(h_o) \approx 1 \text{m/s}$, and $m \approx 70 \text{s}^{-1}$ are roughly constant with $u_\ast$. Inserting Eqs. (4) and (5) in Eq. (1), we obtain,

$$c_v = \frac{\langle v_x^2 \rangle}{\langle v_x \rangle^2} = \frac{\langle v_x^2 \rangle}{\langle v_x \rangle^2} \times \frac{\int_0^\infty \rho dz \int_0^\infty \rho(z) dz}{\langle \rho \rangle \int_0^\infty \rho(z) dz} \approx \frac{\langle v_x^2 \rangle}{\langle v_x \rangle^2} \left( 1 + \left( \frac{m z_p}{\langle \rho \rangle (h_o) + m z_p} \right)^2 \right) \approx 1.17 \frac{\langle v_x^2 \rangle}{\langle v_x \rangle^2} \quad (6)$$

where $\langle v_x \rangle$ and $\langle v_x \rangle(0)$ behave as an approximate estimate for the average cut-off velocity. Using this estimate for $v_c$, Eq.(10) yields,

$$c_v \approx \frac{\int v_x^2 P_v(v_x) dv_x}{\int v_x P_v(v_x) dv_x} \approx 1 + \frac{m z_p}{\langle \rho \rangle (h_o) + m z_p} \approx 1.3. \quad (8)$$

2. Subaqueous regime

We estimate $c_v$ for transport in the subaqueous regime from measurements of the distribution $P_v(v_x)$ of the horizontal velocities $v_x$ of particles in sediment transport under water [4]. These measurements were conducted using particles of average diameter $d = 2.24 \text{mm}$ and under rescaled shear velocity $u_\ast/u_t = 2.1$ [4]. In order to compute $P_v(v_x)$ from the particle trajectories, the particles were considered as being transported if they had a velocity larger than a certain cut-off value, $v_c$ [4]. The distribution of horizontal velocities for the transported particles was fitted using an exponential distribution,

$$P_v(v_x) = \frac{1}{V_f} \exp \left[ -\frac{v_x - v_c}{V_f} \right], \quad (9)$$

where $V_f \approx 110 \text{mm/s}$. By using this distribution, we can compute $c_v$ as,

$$c_v = \frac{\int v_x^2 P_v(v_x) dv_x}{\int v_x P_v(v_x) dv_x} = \frac{1 + \frac{m z_p}{\langle \rho \rangle (h_o) + m z_p}}{\left( 1 + \frac{m z_p}{\langle \rho \rangle (h_o) + m z_p} \right)^2}. \quad (10)$$

Ref. [4] did not report specific values of $v_c$ corresponding to specific measurements of $u_\ast/u_t$ [4]. Instead the authors mentioned that $v_c$ is within the range between $10 \text{mm/s}$ and $30 \text{mm/s}$, depending on the water flow rate. In Ref. [4], $P_v(v_x)$ was obtained using $d = 2.24 \text{mm}$ and $u_\ast/u_t = 2.1$, which correspond to intermediate values for $d$ and $u_\ast/u_t$ investigated in the experiments [4]. Therefore, in order to compute $c_v$ using the horizontal velocity distribution $P_v(v_x)$ obtained in these experiments, we use the intermediate value $v_c = 20 \text{mm/s}$ as an approximate estimate for the average cut-off velocity. Using this estimate for $v_c$, Eq.(10) yields,

$$c_v \approx 1.7, \quad (11)$$

which is the value of steady-state particle speed square correlation for transport in the subaqueous regime.

B. The Coulomb friction coefficient, $\mu$

As mentioned in the main document, the Coulomb friction coefficient $\mu$ is defined by the relation,

$$\langle \rho(v_x v_z) \rangle(0) = -\mu g M/c_a. \quad (12)$$

The left-hand-side of Eq. (12) is the grain shear stress at the bed which is associated with dilute granular flows, $\tau_g = -\langle \rho(v_x v_z) \rangle(0) [7]$. That is,

$$\tau_g = \tau - \tau_{fs}, \quad (13)$$

where $\tau_{fs}$ is the fluid shear stress at the bed. Indeed, different studies showed that, for equilibrium transport ($M = M_s$), $\tau_g$ can be expressed as,

$$\tau_g \approx \tau - \tau_1, \quad (14)$$
whereas this approximation has been verified both for transport in the aeolian regime [2, 8] and for transport in the subaqueous regime [1]. By inserting Eq. (13) into Eq. (12), the following expression for $M_s$ is obtained,

$$M_s = \frac{c_a}{\mu g} \cdot [\tau - \tau_s]. \quad (15)$$

This equation can be then rewritten, using the approximation in Eq. (14), as,

$$M_s \approx \frac{c_a}{\mu g} \cdot [\tau - \tau_s]. \quad (16)$$

Indeed, from experiments on sediment transport in the subaqueous regime [4] it was found, using video-imaging techniques, that $M_s$ behaves according to the expression,

$$M_s = \frac{1}{0.415g} \cdot [\tau - \tau_s]. \quad (17)$$

Therefore, by comparing Eqs. (16) and (17) with $c_a = 1.19$ valid for subaqueous sediment transport (cf. $c_a = 1+1/2s$ with $s = 2.65$), we obtain $\mu/c_a = 0.415$. Thus, in the subaqueous regime of transport, the Coulomb friction coefficient has the approximate value,

$$\mu_{\text{subaqueous}} \approx 0.5. \quad (18)$$

Indeed, values within the range between $\mu/c_a = 0.3$ and $\mu/c_a = 0.5$ — and thus consistent with the value of $\mu$ estimated above — have been reported from measurements of particle trajectories in the subaqueous sediment transport [9–11].

Moreover, for the aeolian regime of transport, the value,

$$\mu_{\text{aeolian}} \approx 1.0, \quad (19)$$

was found in a previous work [2] through determining $\mu$ indirectly both from fitting a sediment transport model which includes Eq. (16) to experimental sediment flux data of Creyssels et al. [3]. We note that $\mu$ is the inverse of the parameter $\alpha$ in Ref. [2].

II. ANALYTICAL EXPRESSIONS FOR CALCULATING THE SATURATED AVERAGE PARTICLE VELOCITY $V_s$

In order to compute the saturation length of sediment transport, the average sediment velocity $V_s$ must be known. We use well-established theoretical expressions reported in the literature to compute $V_s$ in the aeolian regime or in the subaqueous regime.

Aeolian regime — For aeolian transport, we estimate $V_s$ using the model of Ref. [2] since this model showed excellent agreement with measurements of $Q_s$ performed in the experiments of Creyssels et al. [3] — from which we also determined the Coulomb friction coefficient $\mu$ (Section 1B). Ref. [2] gave the following expression for $V_s$,

$$V_s = V_t + \left[3u_t/2\kappa\right] \cdot \ln(V_s/V_t) + \left[u_t/\kappa\right] \cdot F_s(u_t/u_s) \quad (20)$$

and $u_t = \kappa(V_{rs} + V_o) \cdot [(1 - \eta) \cdot \ln(z_{mt}/z_o)]^{-1}$, with,

$$F_s(x) = (1 - x) \cdot \ln(1.78\gamma) + 0.5(1 - x^2) \cdot E_1(\gamma) + 1.154(1 + x \ln(x)(1 - x)^{2.56},$$

$$V_t = V_o(u_t) = V_o + \eta V_{rs}/[1 - \eta]$$

and $z_{mt} = \beta \gamma V_{rs}^{1/4} V_{t}^{2} \cdot [\mu g]^{-1}$, while $V_o = 16.2\sqrt{gd}$, with $g = 1 + 6C/[\tau p d^2]$ [2]. Furthermore, $E_1(x)$ is the exponential integral function, $\kappa = 0.4$ is the von Kármán constant, $\zeta = 5 \times 10^{-4} N/m$ encodes the influence of cohesion [2], $\beta = 0.095, \gamma = 0.17$, and $\eta = 0.1$ are empirically determined parameters [2], and the surface roughness of the quiescent sediment bed is given by $z_o = d \exp(-\kappa B)$, with $B = 8.5 + [2.5 \ln(R_p) - 3] \exp \{-0.11[\ln(R_p)^{2.5}\}$ and $R_p = u_s d/\nu$ [2].

Subaqueous regime — It has been verified in a large number of experimental studies [4, 12–17] that the equilibrium particle velocity in the subaqueous regime of transport approximately follows the expression,

$$V_s = au_s - V_{rs}, \quad (21)$$

where $a$ is a dimensionless number. We note that the above expression is consequence of the equation, $V_s = U_s - V_{rs}$, where $U_s$ is taken proportional to $u_s$. Eq. (21) has been fitted to experimental data assuming $V_{rs} = au_t - b \sqrt{sgd}$, where $b$ is another dimensionless number [4]. This assumption is valid for sufficiently large grain sizes $d$ for which effects of the viscosity $\nu$ on $V_{rs}$ can be neglected. However, the experimental estimates of the saturation length in the subaqueous regime were obtained from experiments using particles of diameters down to four times smaller than the particle size considered in Ref. [4]. Therefore, instead of using the approximation for $V_{rs}$ given above, we take Eq. (6) of the paper to calculate $V_{rs}$. In Fig. 1 we show that using Eq. (21) with the value of $V_{rs}$ computed with Eq. (6) of the paper indeed yields excellent agreement with measurements of $V_s$ reported in Ref. [4]. Moreover, the best fit of Eq. (21) to the experimental data gives $a \approx 4.6$ which is close to the value $a \approx 4.4$ reported in Ref. [4] using the approximated expression for $V_{rs}$ valid for large grain sizes.

III. ESTIMATING THE SATURATION LENGTH $L_s$ FROM THE SIZE OF DUNES

The saturation length of sediment transport can be estimated indirectly from the scale of dunes formed in a given environment. Specifically, two methods can be used.

In the first method, described in Ref. [18], $L_s$ is determined from the minimal cross-stream width of barchan dunes occurring in a given environment. As a matter of fact, barchans which are smaller than a minimal size do not display limbs or slip face and are called domes. The largest dome which has neither slip face nor limbs (and is smaller than the smallest barchan dune with slip face
in the field) indicates the minimal dune size in the field [18]. The cross-stream width of the minimal dune scales with the saturation length as, \( W \approx 12 L_s \) [18].

The second method consists of extracting \( L_s \) from the wavelength (crest-to-crest distance) of the so-called “elementary dunes” [19]. Examples of elementary dunes are those smallest superimposed bedforms occurring on a flat surface or on top of a large barchan dune [20, 21], which may form, for instance, due to a storm wind that makes a small angle with the dominant transport direction [20].

In the following section, we present a summary of the method used to obtain \( L_s \) from the wavelength of elementary dunes. In the subsequent sections, we use this method and the method of the cross-stream width of the minimal barchan in order to estimate \( L_s \) on Earth, Mars, Venus, and under water.

### A. How to estimate \( L_s \) from the wavelength of elementary dunes occurring on the surface of a sediment bed

In this section, we describe how to obtain the saturation length from the wavelength \( \lambda \) of the so-called “elementary dunes” by using the method described in Fourrière et al. [19]. These authors have shown how to compute the spatial shear stress \( \tau(x) \) on top of the transport layer (or, in the absence of transport, on top of the sediment bed) for a flat sediment bed which has a small perturbation \( h(x) \) in the vertical direction. Let \( \tau_0 \) denote the undisturbed shear stress corresponding to \( h(x) = 0 \). Then, the Fourier-transformed shear stress, \( \hat{\tau}(k) \), can be written as [19],

\[
\hat{\tau} = \tau_0 (A + iB) k \hat{h},
\]

where \( k \) is the wavenumber and the “hat” denotes that the Fourier-transformed value of the corresponding quantity is considered. Fourrière et al. [19] used a turbulence model to compute \( A \) and \( B \) (which are both positive numbers) as functions of \( k \) and \( z^*_o \) (the apparent roughness), which is the surface roughness \( z^*_o \) modified due to the presence of the transport layer. The numerical results obtained by the authors can be fitted to [19],

\[
A(R) = 2 + \frac{1.0702 + 0.093069R + 0.10838R^2 + 0.024835R^3}{1 + 0.041603R^2 + 0.0010625R^4},
\]

\[
B(R) = \frac{0.036989 + 0.15765R + 0.11518R^2 + 0.0020249R^3}{1 + 0.0028725R^2 + 0.00053483R^4},
\]

where \( R = \ln \frac{2\pi}{k z^*_o} \). The wavelength \( \lambda = 2\pi/k_{max} \) of the elementary dunes corresponds to the wavenumber \( k_{max} \) under which the dunes grow fastest [19]. By using instability analysis, Fourrière et al. [19] showed that \( \lambda \) is related to the saturation length \( L_s \) through the equation,

\[
\frac{2\pi L_s}{\lambda} = X^{-1/3} + X^{1/3},
\]

where the quantity \( X \) is defined as,

\[
X = -\frac{\hat{B}}{\hat{A}} + \sqrt{1 + \frac{\hat{B}^2}{\hat{A}^2}},
\]

while \( \hat{A} \) and \( \hat{B} \) incorporate dependence on the fluid shear velocity \( \langle u_* \rangle \),

\[
\hat{A} = A(R_{max}) - \frac{\gamma_c A(R_{max}) u_t^2}{1 - \gamma_c u_t^2},
\]

\[
\hat{B} = B(R_{max}) - \frac{\gamma_c B(R_{max}) + \mu_c^{-1} u_t^2}{1 - \gamma_c u_t^2},
\]

In the equations above, \( R_{max} = \ln \frac{2\pi}{k_{max} z^*_o} = \ln \frac{\lambda}{\gamma_c}, \mu_c \approx \tan(32^\circ) \) is the dynamic angle of repose of the sand, and \( \gamma_c \approx 0.5 \) and 0. for the subaqueous and aeolian transport regimes, respectively [19].
In order to estimate $L_s$ using the method described above, knowledge of the threshold shear velocity for sustained sediment transport, $u_t$, as well as the apparent roughness, $z_0^*$, is required. In the Letter, we present the equations used to compute $u_t$ depending on the environmental conditions and the regime of transport. As for $z_0^*$, the equation adopted depends on the transport regime. For the subaqueous regime, it is known that $z_0^*$ does not vary strongly with flow conditions [22], and we thus approximate $z_0^*$ as the surface roughness in the absence of transport ($z_0$). For the aeolian regime, $z_0^*$ is calculated using the analytical model derived in Ref. [2].

We emphasize that the analytical models involved in the estimation of $L_s$ using the method described above potentially introduce significant systematic errors, which could possibly obscure any trend in the data, and which make these estimations of $L_s$ particularly uncertain.

### B. Earth

In Fig. 1 of the paper, we show experimental data corresponding to measurements of the saturation length of aeolian sediment transport under Earth conditions. These measurements are described in the paragraphs which follow.

Andreadetti et al. [23] performed wind-tunnel measurements of the saturation length in aeolian transport under Earth conditions using quartz sand ($\rho_p = 2650$ kg/m$^3$) of average diameter $d = 120$ $\mu$m. The authors measured the sediment flux profiles $Q(x)$ of particles transported over a flat sand bed in the wind tunnel. The flux profiles $Q(x)$ measured at downstream positions ($x$) where the condition $Q(x) > 0.8Q_s$ was fulfilled (i.e. $1 - Q/Q_s < 1$) were fitted using the equation,

$$Q(x) = Q_s \cdot \left[1 - \exp\left(-\frac{x - x_0}{L_s}\right)\right],$$  \hspace{1cm} (29)$$

whereby $L_s$ and $x_0$ were used as fit parameters [23]. The fitted value $L_s$ is the saturation length, since Eq. (29) is a solution of Eq. (1) of the Letter for a flat sand bed ($Q_s(x) = Q_s$). The values of $L_s$ obtained in Ref. [23] in this manner correspond to the brown squares in Fig. 1 of the paper. The error bars associated with the measurements [23] are also displayed in the figure.

Furthermore, indirect estimates of $L_s$ of aeolian transport on Earth were obtained from the wavelength $\lambda$ of elementary dunes on Earth’s dune field by using the method described in Section III A [23]. The sand of the dunes considered in the measurements consisted of quartz particles ($\rho_p = 2650$ kg/m$^3$) of average diameter $d = 185$ $\mu$m [23]. The data corresponding to these estimates are denoted by the green circles in Fig. 1, whereby the error bars denote uncertainties in the measurement of $\lambda$ as described by Andreadetti et al. [23]. Note that, instead of using the method of Ref. [2], the authors estimated $z_0^*$ from empirical fits to the data of Ref. [24]. However, this slightly different method yields a value of $z_0^*$ that is very similar to the one obtained with the equations presented in Ref. [2]. Consequently, approximately the same value of $L_s$ is obtained using either method for estimating $z_0^*$, since the equations presented in Ref. [2] have been shown to agree very well with the data of Ref. [24].

### C. Mars

We estimate the saturation length under Martian conditions from the cross-stream width $W$ of the minimal dune. Ref. [25] reported values of $W$ for two different barchan dune fields on Mars, namely $W \approx 200$ m for the barchan dune field in Arkhangelsky crater, which is located in the southern highlands of Mars, and $W \approx 80$ m for another dune field, which is located near the north pole [25]. From these values of minimal cross-stream width, we obtain $L_s \approx 16.7$ m for the Arkhangelsky barchan dune field and $L_s \approx 6.7$ m for the dune field near the north polar region from $W \approx 12 L_s$ [25].

Furthermore, in order to predict $L_s$ for each dune field using Eq. (2) of the Letter, we need to estimate average density and viscosity of the local atmosphere. Average surface pressure ($P$) and temperature ($T$) values for both fields were obtained from the v23 ARC Mars GCM (see Refs. [26–28] for details), yielding $P = 540$ Pa, $T = 201$ K, and $P = 811$ Pa, $T = 165$ K for the Arkhangelsky crater and north pole dune fields, respectively. From these estimates, we obtain the following average values of fluid viscosity and density: $\nu = 7.23 \times 10^{-4}$ m$^2$/s and $\rho_l = 0.0141$ kg/m$^3$ for the Arkhangelsky crater dune field, and $\nu = 3.11 \times 10^{-4}$ m$^2$/s and $\rho_l = 0.026$ kg/m$^3$ for the dune field near the north pole. On the other hand, both the particle size $d$ of Martian dunes and the typical shear velocity $u_{\text{typ}}$ for which the dune fields were formed are poorly known [8]. As described in the main article, we therefore calculate $L_s$ for a range of particle sizes $d$ ($100 - 600$ $\mu$m [29]) and for two estimates of $u_{\text{typ}}$. The first estimate uses $u_{\text{typ}} = u_t$, consistent with previous studies [18, 30–32], and we derive the second estimate of $u_{\text{typ}}$ below, using the wind speed probability distribution measured at the Viking 2 landing site [33, 34].

1. Estimating $u_{\text{typ}}$

In the following, we estimate $u_{\text{typ}}$ by first obtaining the probability distribution of wind shear velocities at the Viking 2 lander site from measurements [33, 34]. Due to the scarcity of wind speed measurements on the red planet, we then assume out of necessity that this wind shear velocity probability distribution occurs at the Arkhangelsky and north pole dune fields. Using expressions for (i) the saturated mass flux of saltating particles, $Q_s$ [2], as a function of $u_t$ and (ii) the probability $P_t$ that saltation occurs when $u_t < u_s < u_{\text{fl}}$, we obtain estimates of $u_{\text{typ}}$. 


We start our approach by defining $u_{*\text{typ}}$ as the saltation-flux weighted average of $u_*$ that is,

$$
\begin{align*}
    u_{*\text{typ}} &= \frac{\int_0^\infty u_* P_{u_*} P_{tr}(u_*) Q_s(u_*) du_*}{\int_0^\infty P_{u_*} P_{tr}(u_*) Q_s(u_*) du_*},
\end{align*}
$$

where $P_{u_*}$ is the probability of occurrence of winds with shear velocity $u_*$, $P_{tr}$ is the probability that saltation is occurring for a given $u_*$, and $Q_s$ is the equilibrium sand transport rate (provided saltation is occurring) for a given value of $u_*$. We derive expressions for each of these functions below.

2. The probability distribution of the wind shear velocity

The probability distribution of wind speeds is commonly described using the Weibull distribution $[35]$, given by,

$$
    P(U) = \frac{k}{c} \left( \frac{U}{c} \right)^{k-1} \exp \left[-\left(\frac{U}{c}\right)^k\right],
$$

where $U$ is the wind speed at a given height $z$, $k$ is a dimensionless shape parameter, and $c$ is a scale speed that is proportional to the average wind speed through,

$$
    c = \frac{\bar{U}}{\Gamma(1 + 1/k)}.
$$

Here, $\bar{U}$ is the wind speed averaged over much longer time periods than $U$; typically, $U$ is averaged over several minutes to an hour, and $\bar{U}$ is averaged over one or several years. The long-term averaged $\bar{U}$ can be related to the long-term averaged shear velocity $\bar{u}_*$ through the “law of the wall” $[36]$. That is,

$$
    \bar{U} = \frac{\bar{u}_*}{\kappa} \ln \left( \frac{z}{z_0} \right),
$$

where $z_0$ is the aerodynamic roughness length, and $\kappa \approx 0.40$ is the von Kármán constant. Combining the above equations then yields the probability distribution of $u_*$,

$$
    P(u_*) = \frac{k}{c_{u_*}} \left( \frac{u_*}{c_{u_*}} \right)^{k-1} \exp \left[-\left(\frac{u_*}{c_{u_*}}\right)^k\right],
$$

where,

$$
    c_{u_*} = \frac{\kappa c}{\ln(z/z_0)}.
$$

We obtain the wind speed scaling parameters $c$ and $k$ from the only long-term data set of Martian wind speeds: the measurements made by the Viking 2 lander over a period of 1040 sols $[33]$. We thus use $c = 3.85$ m/s and $k = 1.22$ m/s as calculated by Ref. $[34]$. Moreover, we use $z_0 = 1$ cm after Ref. $[37]$.

3. The saltation transport probability $P_{tr}$

Predicting the sand transport rate for a given value of $u_*$ is complicated by the occurrence of hysteresis in Martian saltation $[8, 38]$. That is, recent studies have estimated that the value of $u_*$ for which saltation is initiated ($u_{tr}$) exceeds the minimum value of $u_*$ for which saltation can be sustained ($u_t$) by up to a factor of $\sim 10$ $[8, 38]$. Whether or not saltation is occurring for a given value of $u_*$ intermediate between $u_t$ and $u_{tr}$ thus depends on whether the wind speed exceeded $u_{tr}$ more recently than that it dropped below $u_t$. By also assuming that the probability distribution of martian wind speeds can be described by a Weibull distribution, Ref. $[38]$ derived an equation estimating the probability that transport occurs for values of $u_*$ intermediate between $u_t$ and $u_{tr}$,

$$
    P_{tr} = \frac{\exp\left\{-[u_t \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\} - \exp\left\{-[u_{tr} \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\}}{1 - \exp\left\{-[u_t \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\} + \exp\left\{-[u_{tr} \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\}},
$$

where $u_{tr}$ is calculated following Ref. $[39]$. Of course, when $u_* < u_t$ and $u_* > u_{tr}$, we have that,

$$
    P_{tr} = 0, \quad (u_* < u_t),
$$

$$
    P_{tr} = 1, \quad (u_* > u_{tr}).
$$

4. The estimated $u_{*\text{typ}}$ and $L_s$

We can now estimate $u_{*\text{typ}}$ by calculating the mass flux $Q_s$ using Eq. (69) in Ref. [2], and inserting this together with Eqs. (34)–(38) into Eq. (30). The resulting estimate of $u_{*\text{typ}}$ is plotted in Fig. S 2a as a function of particle size at both dune fields. Since Martian wind speeds rarely exceed $u_{tr}$, at least at the Viking 2 landing site $[33, 34]$,
we find that \( u_{styp} \) is closer to \( u_t \) than to \( u_h \). The resulting saturation length \( L_s \) predicted with these values of \( u_{styp} \) and Eq. (2) of the main article is plotted in Fig. S 2b.

D. Subaqueous dunes

By using the method described in Section III A, Fourrière et al. [19] estimated the saturation length of subaqueous transport from measurements of the wave-length \( \lambda \) of elementary transverse bedforms produced in water flumes [40–42]. These experiments were performed using different types of granular materials, namely natural sand particles or glass beads. The data corresponding to the estimates of \( L_s \) by Fourrière et al. from these experiments are represented by the blue (for natural sand particles) and green (for glass beads) symbols in Fig. 2 of the paper.

Furthermore, we have estimated \( L_s \) for subaqueous transport from the minimal cross-stream width \( W \) of subaqueous barchans produced in the experiments by Franklin and Charru [43]. From these measurements, we obtain \( L_s \) from the relation, \( L_s = W/12 \). The red symbols in Fig. 2 denote the values of \( L_s \) estimated in this manner.

E. Venus

We estimate the saturation length of sediment transport on Venus from the wavelength of microdunes produced in wind tunnel experiments by Marshall and Greeley [44] mimicking the Venustian atmosphere.

By adjusting the pressure and the temperature in the wind tunnel, Marshall and Greeley [44] obtained values of air viscosity (\( \nu = 2.9 \times 10^{-7} \text{ m}^2/\text{s} \)) and air density (\( \rho_f = 52.94 \text{ kg/m}^3 \)) similar to those occurring in the atmosphere of Venus. Experiments using natural sand (quartz particles) of average diameter \( d = 150 \mu \text{m} \) under free stream velocity \( U_\infty \approx 0.8 \text{ m/s} \) produced microdunes of wavelength \( \lambda \approx 20 \text{ cm} \) (cf. Fig. 7 of Ref. [44]). From Fig. 2 of Ref. [44] we obtain the threshold free stream velocity \( U_\infty \) corresponding to the grain diameter \( d = 150 \mu \text{m} \) used in the experiments, that is, \( U_\infty \approx 0.6 \text{ m/s} \). From the value of \( U_\infty \), we can estimate the shear velocity ratio \( u_\ast/u_t \) in leading order, \( u_\ast/u_t \approx U_\infty/U_\infty \approx 1.3 \). By taking this value of \( u_\ast/u_t \) and using the method described in Section III A, we estimate \( L_s \approx 6 \text{ mm} \) from the wavelength of the Venustian microdunes produced in the wind tunnel. We display this indirect estimate of \( L_s \) in Fig. 3 of the paper.

IV. A SIMPLE EXPRESSION TO COMPUTE \( L_s \) IN THE AEOLIAN REGIME

We have shown in the paper that the expression of \( L_s \) obtained for the aeolian regime of transport (Eq. (10) of the paper) can be written in an approximated form as,

\[
L_s^{\text{aeolian}} \approx 3 c_v V_s^2 \cdot [\mu g]^{-1}.
\]

The advantage of this approximation is that it provides a simpler expression for computing \( L_s \) as a function of the steady-state particle velocity \( V_s \) in the aeolian regime. In this section we discuss the performance of this approximated expression in comparison to the original expression ((Eq. 10) of the paper).

---

Fig. S 2. (a) The estimated typical shear velocity \( u_{styp} \) for which bedforms are formed on Mars (dash-dotted lines) for the dune fields at Arkhangelsky crater (brown lines) and the north pole (blue lines). Also shown are the fluid threshold \( u_\theta \) at which sediment transport is initiated (solid lines) and the impact threshold \( u_i \) below which transport cannot be sustained (dashed lines). (b) Saturation length predicted with Eq. (2) and \( u_{styp} \) derived from Eq. (30) (dash-dotted lines) and from \( u_{styp} = u_\theta \) (solid lines). Also shown are the values of \( L_s \) estimated from the minimal size of barchan dunes (dashed lines).
The approximated expression (Eq. (39)), resulted from approximating the feedback term \( K \) (Eq. (9) of the paper) as its limit for large dimensionless shear velocities \((u_*/u_t)^2 \gg 1\). Consequently, the approximation performs best in the range of large values of the ratio \( u_*/u_t \), as we can see in Fig. 3. This figure shows \( L_s/(sd) \) as a function of \( u_*/u_t \) and different values of \( d \) for Earth (blue lines) and Mars (red lines) conditions using the original expression for \( L_s \) in the aeolian regime (Eq. (10) of the paper; continuous lines) and the approximated expression (Eq. (39); dotted lines). We note that for all conditions, the deviation increases as \( u_* \) approaches \( u_t \). Indeed, we see in Fig. 3 that, for typical conditions where the particle size is between 200\( \mu m \) and 500\( \mu m \) [8, 29, 45], the predictions of \( L_s \) from both expressions, original and approximated, deviate from each other by less than 30%.

Fig. S 3. Dimensionless saturation length, \( L_s/(sd) \), as a function of the dimensionless shear stress, \( u_*/u_t \), for aeolian transport. Shown are predictions with different values of \( d \) for Earth (blue lines) and Mars (red lines) conditions using the original expression for \( L_s \) in the aeolian regime (Eq. (10) of the paper; continuous lines) and the approximated expression (Eq. (39); dotted lines).


