Simulations of binary collisions of barchan dunes: the collision dynamics and its influence on the dune size distribution

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Large dune fields can overrun for instance infrastructures lying in the way of their motion. Unidirectional wind fields and low sand availability give rise to single barchan dunes moving in the direction of wind. They can interact by directly exchanging their sand through collisions. This kind of interaction plays a crucial role in the evolution of barchan dune fields, for instance on the selection of a characteristic dune size. Simulations of dune collisions with lateral offset are studied systematically and described by general phenomenological rules. Moreover, simulations with a simplified model without sand flux considering only collisions show that the sizes of sand dunes in such a scenario follow a Gaussian distribution with a well defined characteristic size.

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I. INTRODUCTION

Vast areas of our planet have arid environments where non–living matter predominates. The rare vegetation allows at many places the existence of highly mobile sand dunes which move along the predominant wind direction up to several decades of meters per year. For instance, in

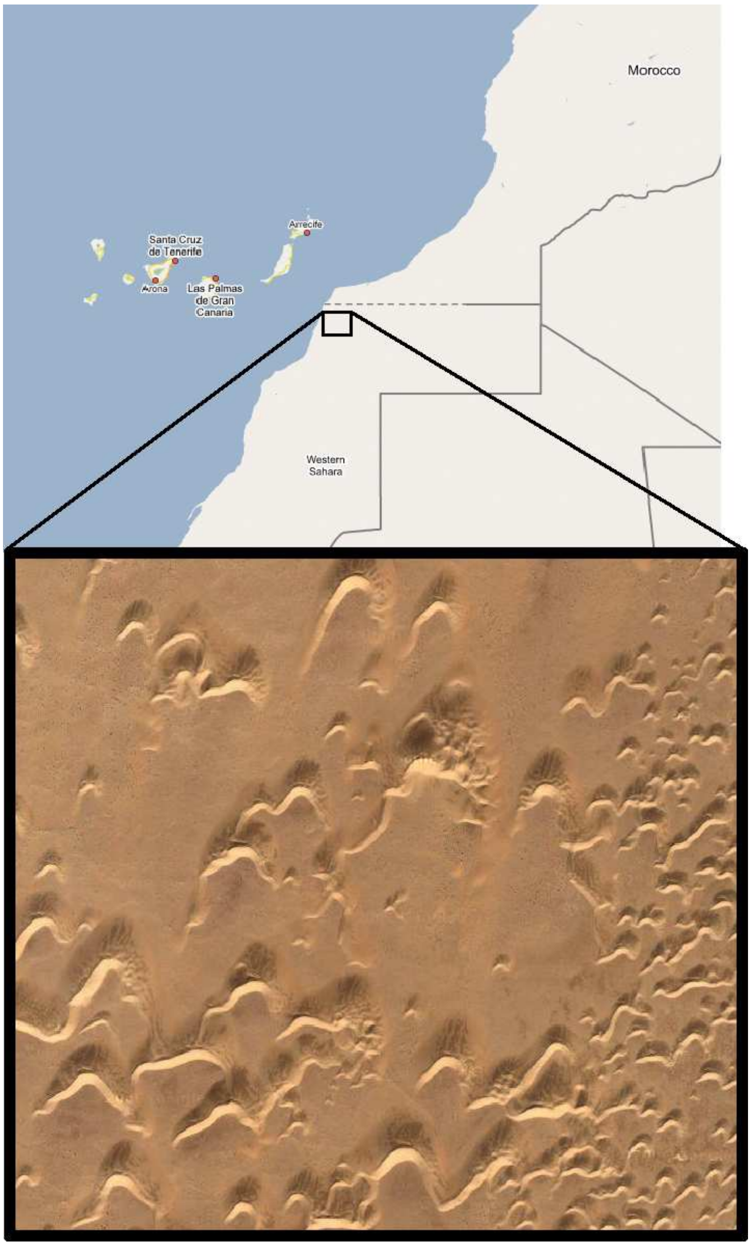

FIG. 1: Example of a barchan dune field in Morocco, West Sahara. Note that the dune field is divided in corridors along the wind direction, where the dune size is roughly uniform.

Morocco at the latitude of the Canarian Islands sand is shifted by wind from the beaches hundreds of kilometers into the continent (see Fig. 1). The unidirectional wind regimes and not too high sand coverage lead to the formation of a particular type of sand dunes called barchans. Barchan dunes don't appear isolated, instead, they belong to several kilometers long dune fields forming corridor structures oriented along the wind direction (Fig. 1). The intrinsic instability of barchan dunes under an incoming sand flux leads to an increase of the largest dunes in the field whereas

the smaller ones shrink until they disappear [1–3]. Hence, the mean size of the dunes should grow with the distance from the beginning of a field. Nevertheless, in many dune fields sizes saturate (Fig. 1). Two mechanisms have been proposed to avoid unlimited dune growth: instability of large dunes due to changing wind directions [4] and collisions between dunes [2, 5–7].

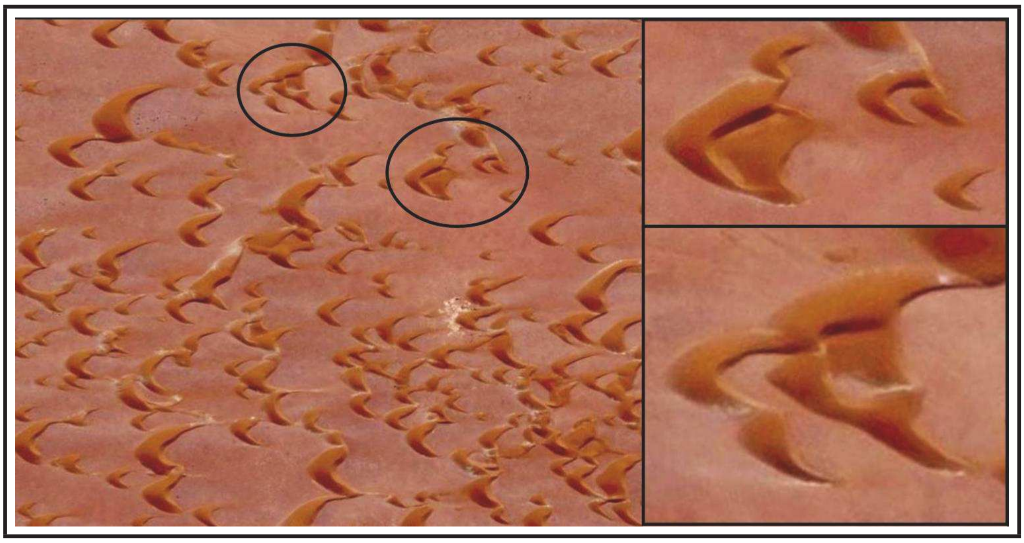

FIG. 2: Collisions between barchan dunes are ubiquitous in a dune field. Here are some examples of collisions in a Moroccan dune field.

Here, we focus on the case when a dune collides with another one. Collisions are ubiquitous in dune fields (Fig. 2) due to the relatively broad range of different velocities, which are inversely proportional to the dune size. Due to this velocity-size dependence, the dune that collides onto a second one must be smaller than the latter. Some indications of collision processes have been observed several times [8, 9] but their dynamics was not understood until recently [2, 5–7]. The large temporal scales of such a process makes it difficult to observe the final state after a collision. Simulations using a minimal dune model are carried out to understand what happens if two dunes collide with each other and different results are obtained depending on the relative volume difference between the interacting dunes [5] and the relative offset between their centers. As will be shown, the three different final situations of coalescence, solitary wave behavior and breeding provide an effective mechanisms to redistribute sand and thus to avoid the continuous growth of dunes in a dune field. A result also confirmed by under-water experiments of binary collisions of barchan dunes [2].

Fro the simulations of dune collisions with lateral shift we extract a general rule that fits the resulting volumes. This collision rule is implemented into a model for the evolution of a simplified

dune field where dunes randomly collide with each other. From this model we found that collisions act as a mechanism for the redistribution of sand, and thus, they are capable of selecting a characteristic dune size. In fact, dune sizes in such a scenario follow a Gaussian distribution.

A. Collision dynamics

This section presents the results of the minimal dune model that takes into account the calculation of the turbulent wind field, the sand flux and the height changes on the surface. The minimal dune model considers the height profile of sand dunes in a unidirectional wind field and calculates its impact on the air flow and the sand flux over the dune. As a result of the modified sand flux the dune shape may change and the whole process repeats itself until eventually a final shape stationary shape is obtained. The model is explained in detail in refs. [3, 10–12].

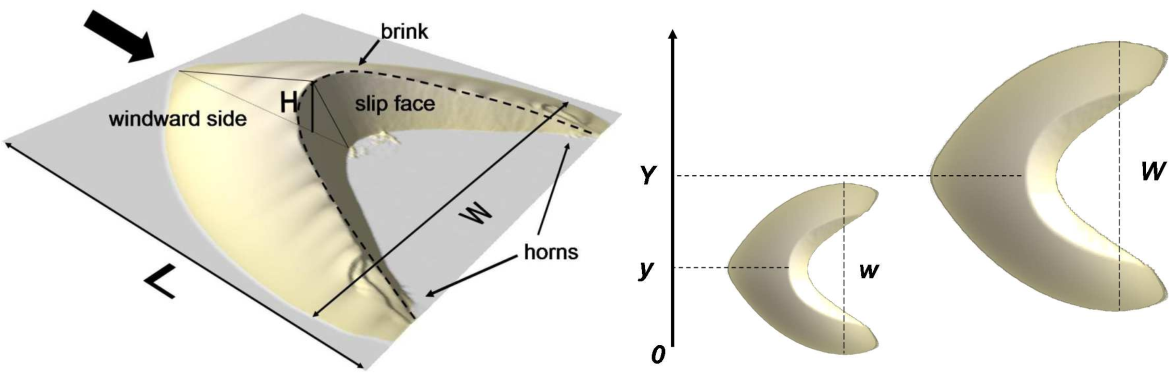

FIG. 3: Left: example of a barchan dune with its main morphological characteristics. The arrow shows the prevailing wind direction. Right: Sketch of the initial state of a binary collision between two barchan dunes of sizes w and W, and centered at y and Y, respectively.

Simulations with the dune model are carried out to obtain the outcome of collisions between two dunes for different initial lateral offsets θ_i and volume ratios r_i keeping the volume of the larger barchan constant at 6000 m^3. We define the relative lateral offset between two dunes as $\theta \equiv \frac{Y-y}{W/2}$, where Y and y are the coordinates of the crest of the large and the small dune in the lateral direction transverse to their movement, respectively, and W is the width of the larger dune. Notice that θ is constructed to be larger than 1 if the crest of the smaller dune don't cross the larger dune (Fig. 3). Besides, the volume ratio is defined as $r \equiv \frac{v}{V}$, where V is the volume of the large barchan and v the volume of the small one. The strength of the wind blowing into the system is

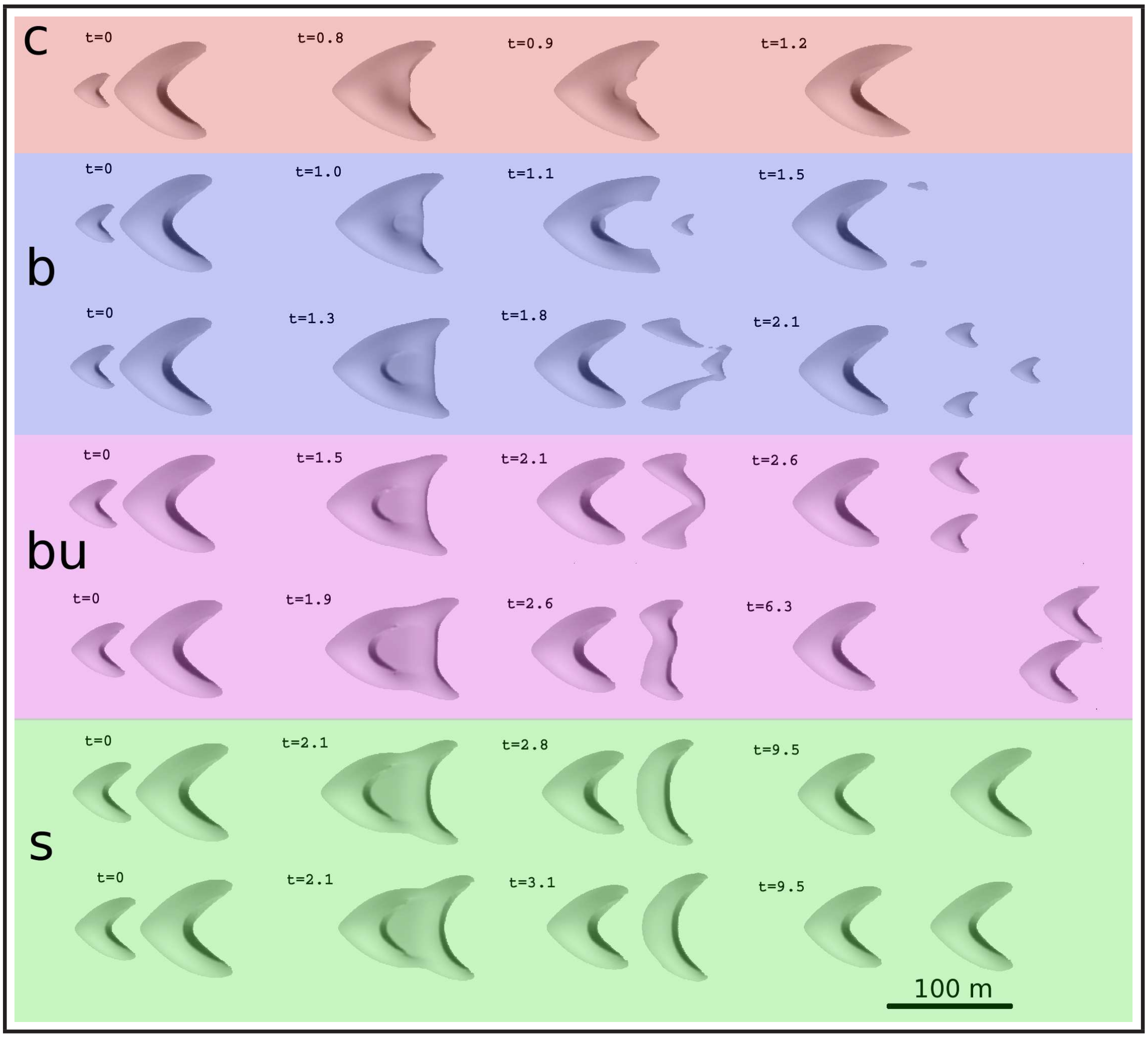

FIG. 4: Snap shots of the time evolution (from left to right) of the collision of two barchans for zero lateral offset and volume ratios: $r_i = 0.06$, 0.08, 0.12, 0.17 and 0.3. The volume ratio increases from top to bottom. Depending on the volume ratio, coalescence (c), breeding (b), budding (bu) and solitary wave behavior (s) take place. Wind blows from left to right. The time (t) is in years, taking into account that wind blows continuously during all the year. This figure is from [5].

fixed to a shear velocity of 0.5 m/s. Open boundary conditions are used with an influx of 20 m^2/yr, equal to the big barchan equilibrium out-flux [5].

Figure 3 (right) shows a sketch of the initial condition used for the simulation. Based on the linear relations between dune height, width, length (Fig. 3) and the cubic relation between width and volume, as observed in both measurements [13–15] and models [1, 3, 16], the morphology of a barchan dune can be approximately characterized solely by its width w (Fig.3). Therefore, as we also found from the simulations, the collision outcome is mainly determined by the width (or

volume) and the offset of the colliding dunes.

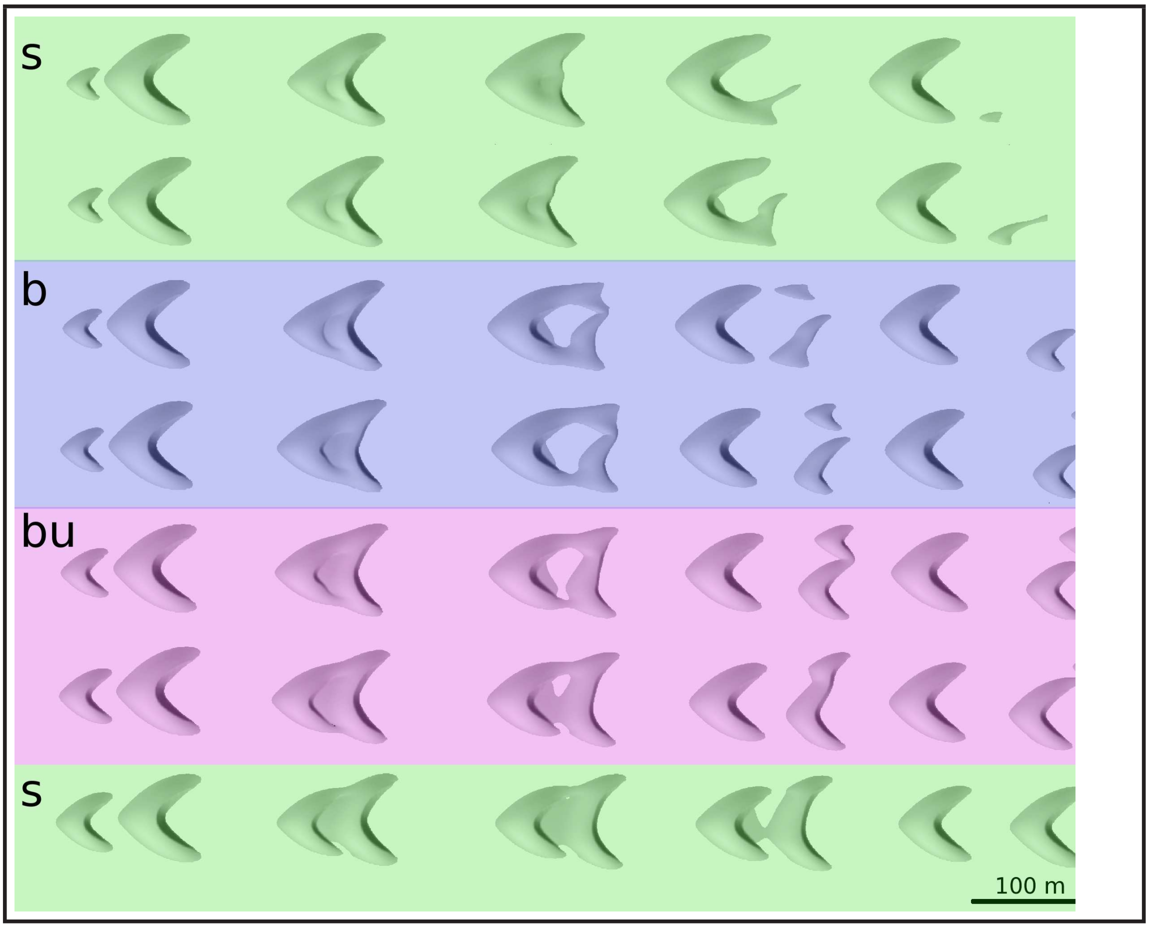

FIG. 5: Snap shots of the time evolution of binary collision for $\theta_i = 0.1$ and volume ratios (from top to bottom): $r_i = 0.06$, 0.08, 0.12, 0.17 and 0.3. Letters and colors distinguish the different results after collision.

Figures 4, 5, 6, 7, 8 and 9 illustrate the evolution of the collision process for different volume ratios and increasing offsets, starting from zero. After the smaller barchan bumps at some point onto the larger one, a hybrid state is formed where the two dunes melt into a complex pattern. Depending on the initial relative size r_i and their lateral offset θ_i, four different situations can emerge after collision: coalescence (Fig. 4, ‘c’) where only one dune remains, breeding (Fig. 4, ‘b’) and budding (Fig. 4, ‘bu’) where two dunes leave the larger one, and solitary wave behavior (Fig. 4, ‘s’) where the number of dunes remains two after the collision.

These four situations were already studied in detail for axial (zero offset) collisions in [5]. Therefore, here we will only describe them by including the modifications in the collision output due to a finite offset, but without entering into specific details. For small relative volume r_i and

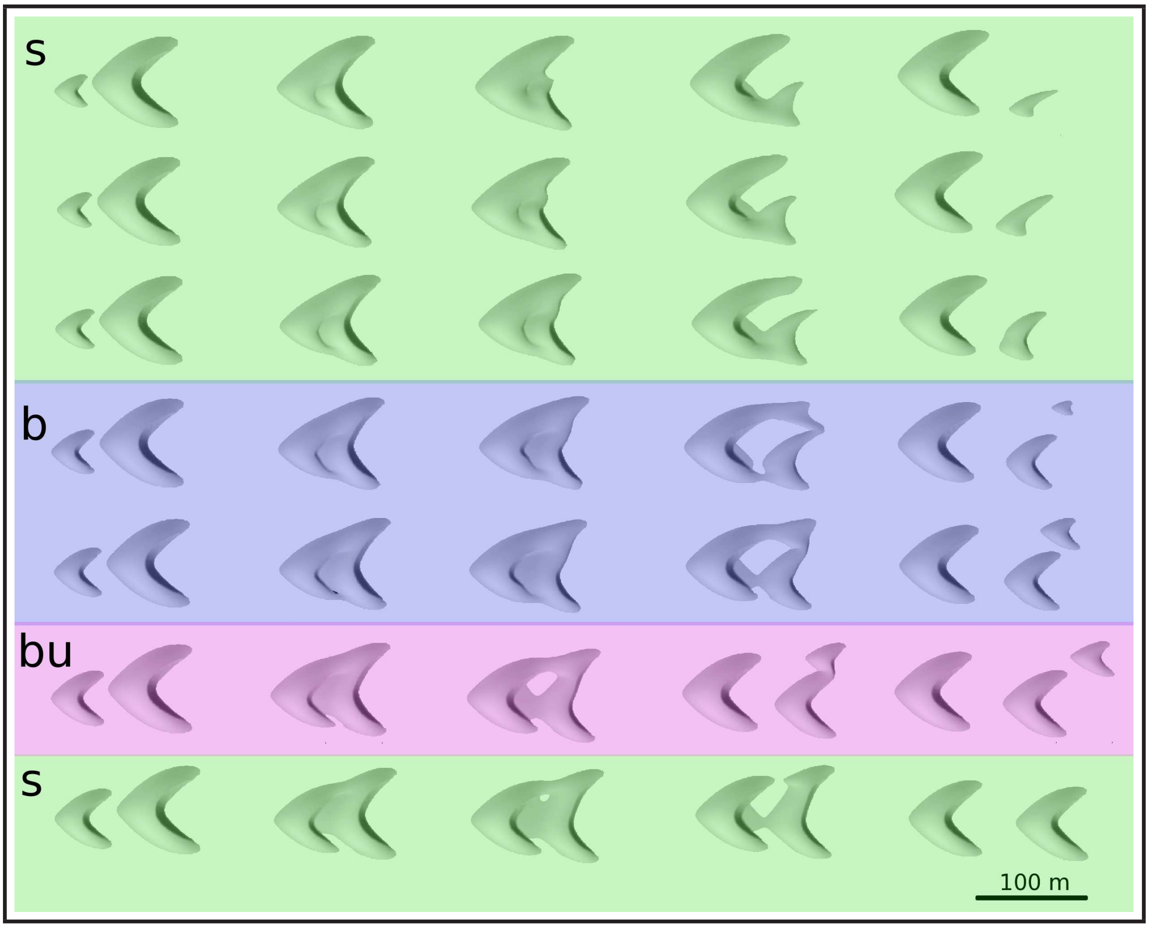

FIG. 6: Snap shots of the time evolution of binary collision for $\theta_i = 0.2$ and volume ratios (from top to bottom): $r_i = 0.06, 0.08, 0.12, 0.17$ and 0.3. Letters and colors distinguish the different results after collision.

lateral offset θ_i both dunes coalesce to a single one (red 'c' zone in Fig. 4). For a larger r_i and θ_i, a small barchan is ejected from the hybrid state, in what we call solitary wave behavior (upper green 's' zone in Figs. 5, 6, 7, 8 and 9).

For an even larger r_i and only for a given range of the lateral offset, the surface perturbation created by the incoming small dune on top of the large one propagates over the dune horns (Fig. 3). As a result, at the end of one or both horns, depending on the offset, a small dune is also ejected besides the main ejected dune. This phenomenon of double ejection, the ejection of a dune from the dune body and another small one from the horn, we call 'breeding' (blue 'b' zone in Figs. 4, 5, 6, 7 and 8). Collisions in this regime show a qualitative similarity with the barchan field shown in Fig. 2.

As can be observed from most simulations with finite offset, in the case of 'breeding', one of

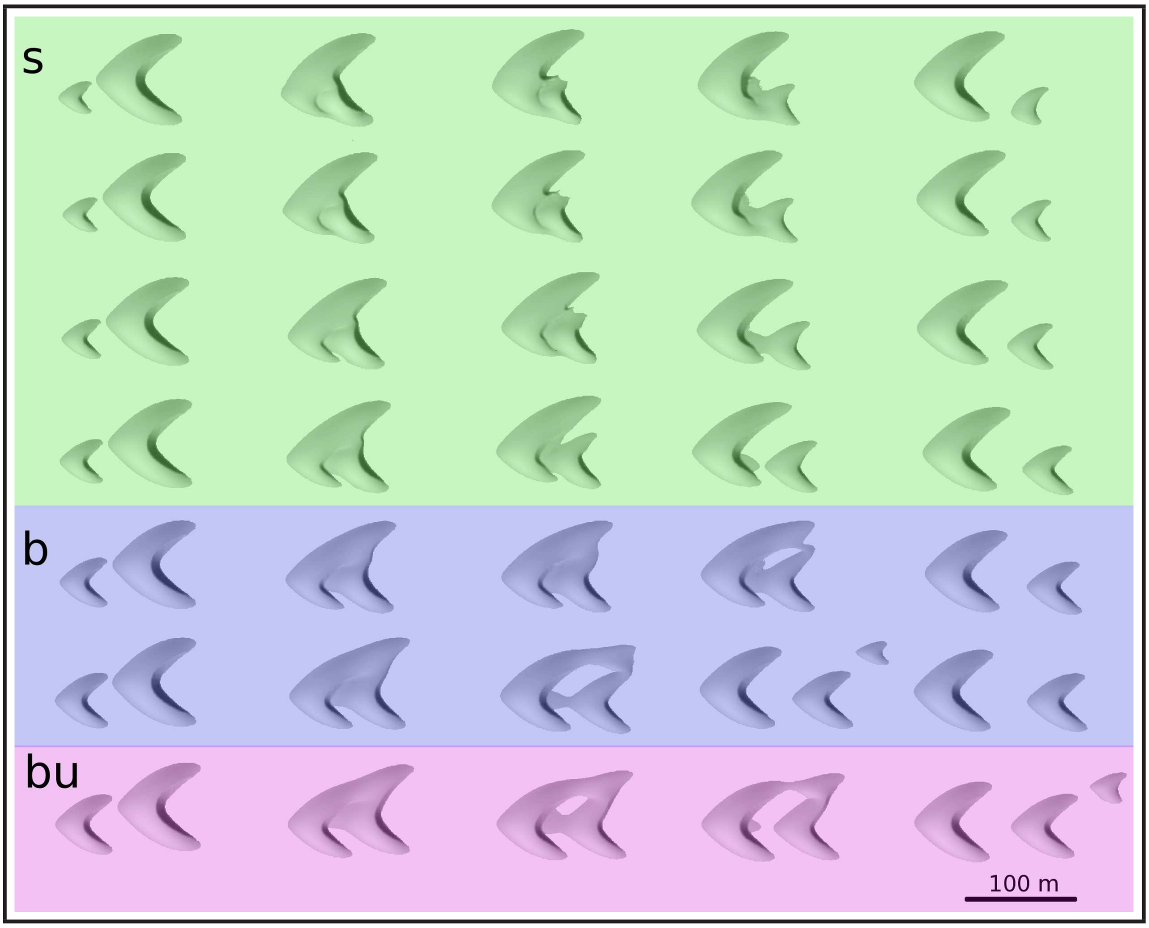

FIG. 7: Snap shots of the time evolution of binary collision for $\theta_i = 0.4$ and volume ratios (from top to bottom): $r_i = 0.06$, 0.08, 0.12, 0.17 and 0.3.

the two dunes that emerge from the horn is very small compare to the other one. Since this dune generally is not stable, we will neglect it and consider the output of the collision as consisting only of two dunes, one that leaves the hybrid state and a large one that remains behind. Therefore, in the following breeding will be equivalent to solitary wave behavior.

As the initial volume ratio increases, the smaller relative velocity between the dunes reduces their overlapping, the hybrid state is not really formed and both dunes separate again. However, the leaving dune is unstable and splits into two new dunes, a phenomenon we call 'budding' (pink 'bu' zone in Figs. 4, 5, 6 and 7). A similar phenomena was reported in experiments with sub-aqueous barchans [2, 7].

For a larger volume ratio and small lateral offset, the instability of the leaving dune disappears and it develops into a full barchan. Then, we observe again solitary wave behavior as is shown in Fig. 4(s) and the lower region 's' of Figs. 5 and 6. In contrast with the solitary wave behavior at

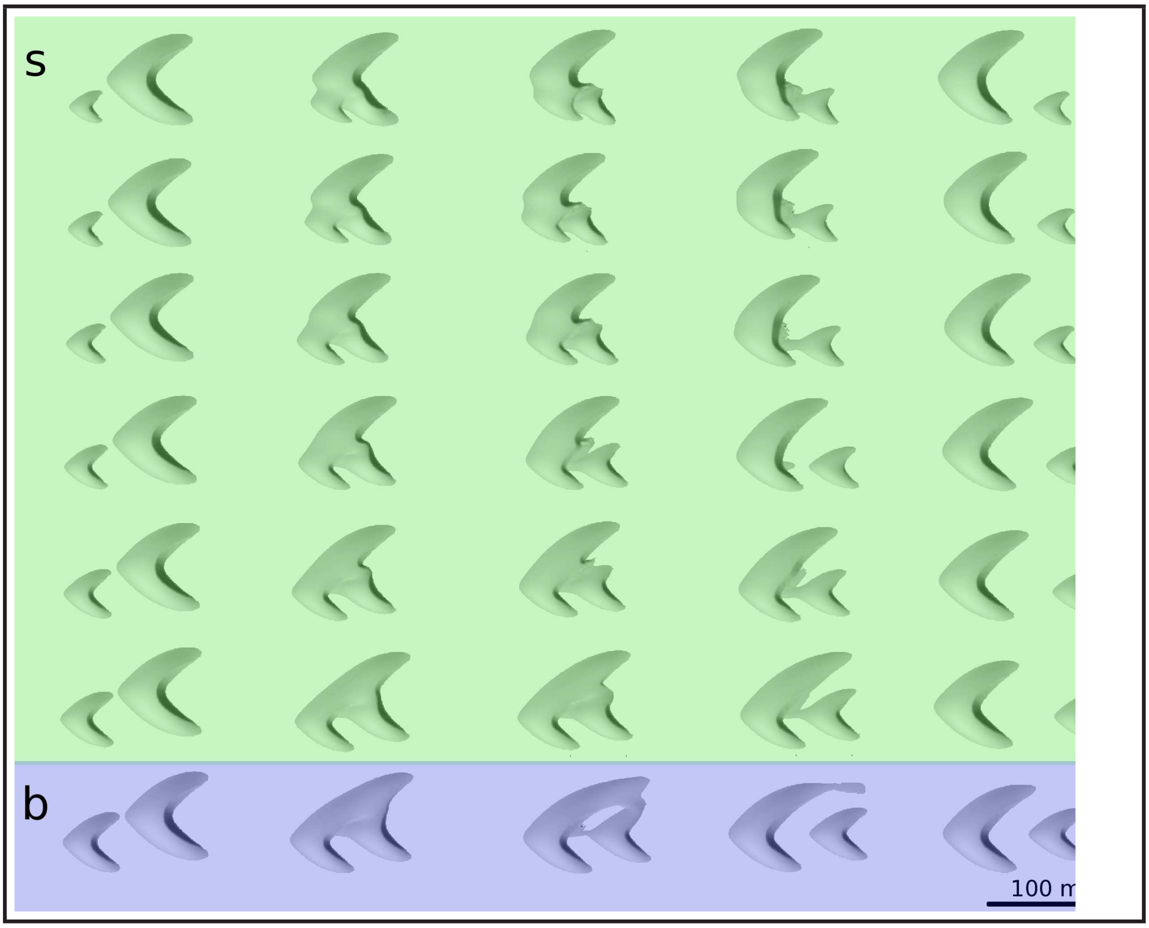

FIG. 8: Snap shots of the time evolution of binary collision for $\theta_i = 0.6$ and volume ratios (from top to bottom): $r_i = 0.06, 0.08, 0.12, 0.17$ and 0.3. Notice that only breeding (b) and solitary wave behavior (s) appear.

small volume ratios or large offsets (as explained above), in this case, the sizes of the dunes after collision is very similar.

II. BINARY COLLISION PHASE DIAGRAM

The morphological phase diagram of binary collisions is schematically showed in Fig. 10 in terms of the final volume ratio r_f as a function of the initial volume ratio r_i and lateral offset θ_i. A similar picture, although less rich, was also numerically obtained in [17]. In order to include breeding and budding cases where more than one dune leaves the hybrid state the final volume ratio is defined as ratio between the volume of the total outgoing dunes and the remaining one.

From the fit of the numerical simulations data, the volume ratio r_f after a collision can be

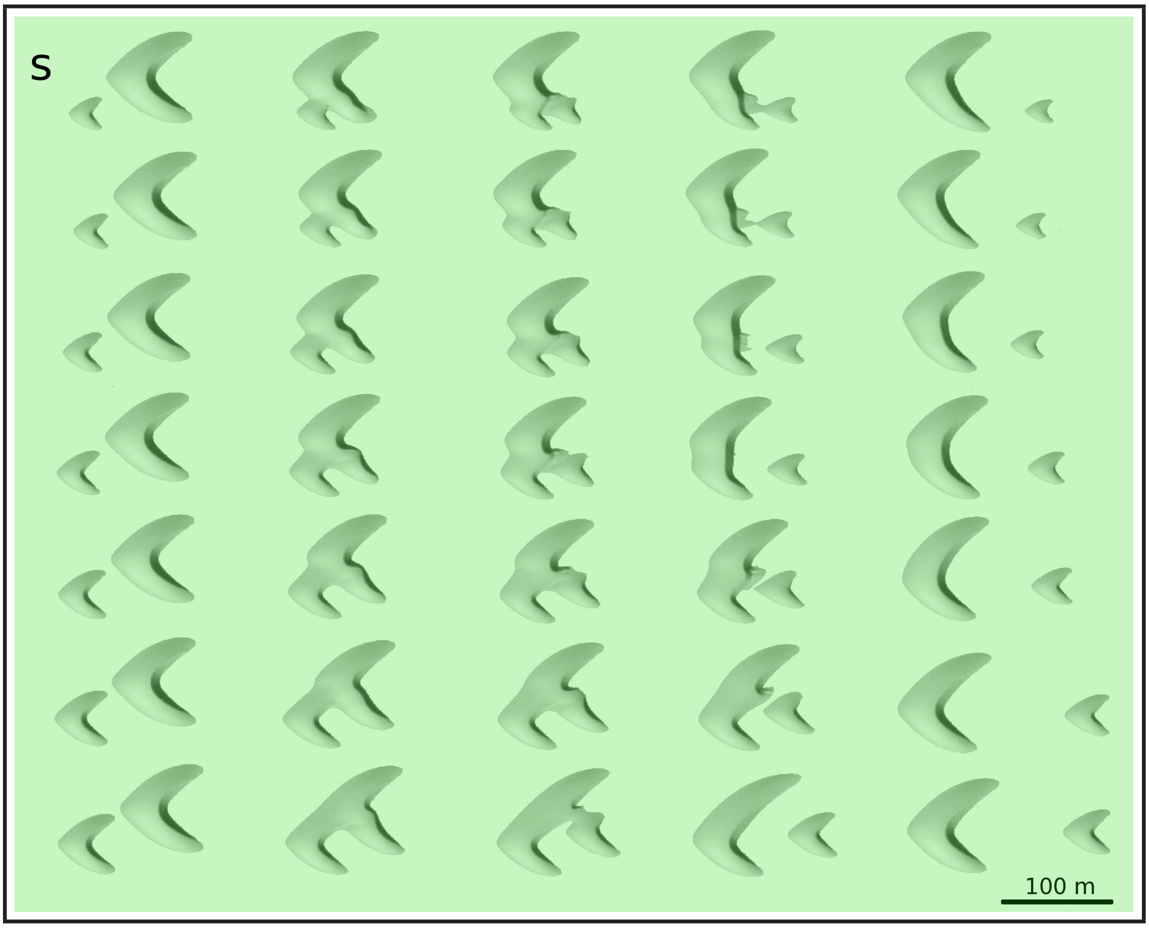

FIG. 9: Snap shots of the time evolution of binary collisions for $\theta_i = 0.8$ and volume ratios (from top to bottom): $r_i = 0.06$, 0.08, 0.12, 0.17 and 0.3, where only solitary wave behavior (s) occurs.

expressed by the phenomenological equation,

$$r_f(\theta_i, r_i) \approx 1 - \exp[-A(\theta_i)(r_i - r_0(\theta_i))^{4/3}] \tag{1}$$

valid for $r_0 < r_i < 1$. This condition takes into account that there is a minimal relative size r_0 for the incoming dune below which no dune comes out, i.e coalescence occurs (red zone in Fig. 10). The coalescence threshold r_0 is function of the initial lateral offset θ_i, and is given by,

$$r_0(\theta_i) \approx 0.12\ \exp[-(\theta_i/0.4)^2] - 0.05 \tag{2}$$

This equation also defines a maximum offset $\theta_i \sim 0.4$ above which no coalescence occurs. Furthermore, in the range $r_i \geq 1$, dune sizes are barely affected by the collision and the trivial relation $r_f = r_i$ immediately follows.

On the other hand, the term $A(\theta_i)$ represents the sensibility of the final volume ratio r_f to the

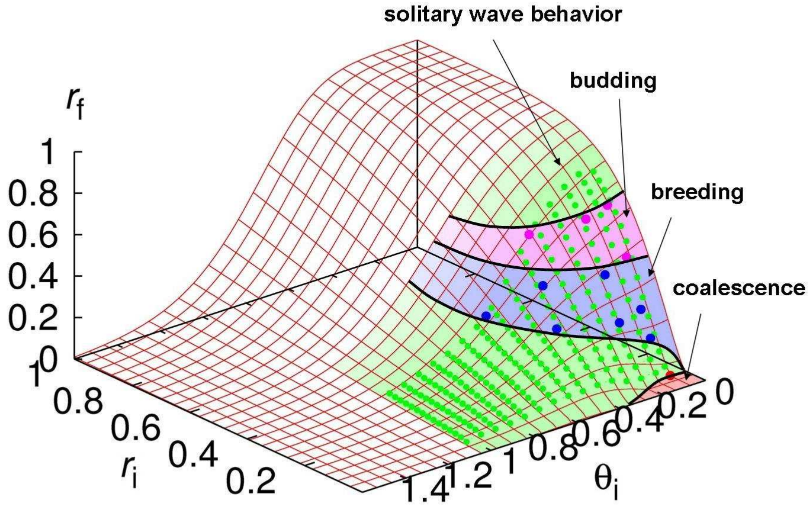

FIG. 10: Sketch of the morphological phase diagram for binary collisions. The volume ratio r_f after the collision is plotted as a function of the initial offset θ_i and the initial volume ratio r_i. Dots represent simulation results.

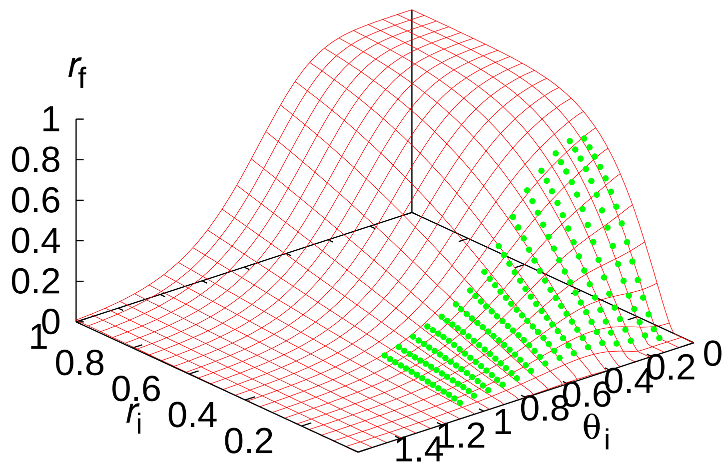

FIG. 11: Volume ratio r_f after the collision as a function of the initial offset θ_i and the initial volume ratio r_i. Dots show the measured values obtained from the simulations.

initial offset θ_i,

$$A(\theta_i) \approx 10 \exp[-(\theta_i/0.36)^{4/3}] \, . \tag{3}$$

Additionally, the lateral positions of both dunes change after the collision. However, no simple

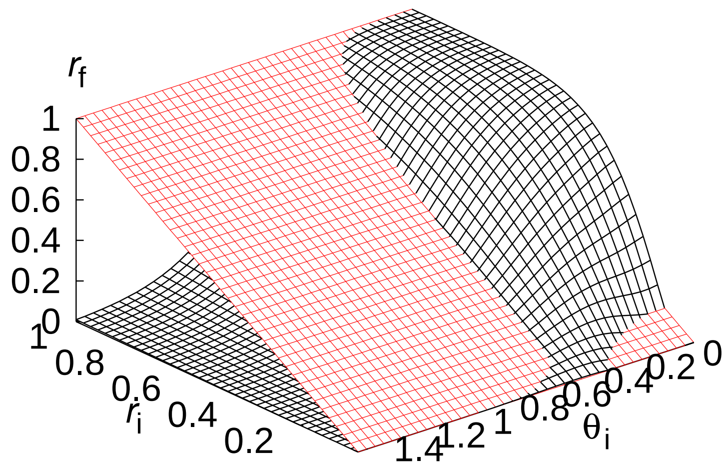

FIG. 12: Volume ratio r_f after the collision as a function of the initial offset θ_i and the initial volume ratio r_i. The plane $r_f = r_i$ is also plotted in order to show the regions of phase space where the total sand volume is redistributed between both dunes after collision ($r_f > r_i$) or is accumulated by the bigger one ($r_f < r_i$).

rule could be found and thus we do not consider the final offset in the collision rule. Figure 11 illustrates the shape of $r_f(r_i, \theta_i)$ as compared to the data obtained from the numerical simulations.

An important consequence of the collision process is that it changes the volume distribution of the dunes in a barchan field. Figure 12 shows the volume ratio after the collision as a function of the initial volume ratio and lateral offset, along with a plot of the plane $r_f = r_i$. Since their sizes are not constant in time, due to the permanent exchange of sand between the dunes and its intrinsic instability [1], we use the volumes of the dunes immediately before and after they leave the hybrid state. As a result of the interaction, the relative volume strongly increases for $\theta_i < 0.6$ and most of the initial volume ratios, i.e. $r_f > r_i$. In this case the collision process redistributes the initial mass making both dunes more similar, giving rise to a size selection mechanism in barchan fields [1]. Whereas for offsets larger than 0.6, $r_f < r_i$, the bigger dune before collision increases its size further by taking mass from the smaller dune.

III. DUNE SIZE DISTRIBUTION EMERGING FROM THE COLLISION DYNAMICS

A. Flux Balance on a single Barchan Dune

Based on the results of numerical simulations (see ref. [3]), if we consider dune shapes as scale invariant in a first approximation, the flux balance of a barchan dune of volume V and width w obeys the following relation

$$\dot{V} \propto w\,Q\,\left(\frac{q_{in}}{Q} - \frac{q_c}{Q}\right) \tag{4}$$

where Q is the undisturbed saturated sand flux on a plane, q_{in} is the dune influx per unit length and $q_c \approx 0.18\,Q$ denotes the equilibrium influx at which the dune volume does not change [3]. However, this equilibrium is unstable since there is no mechanism by which barchan dunes can change their out-flux to adjust it to a given influx.

In general the flux balance on a dune is not in equilibrium. However, assuming that the flux inside the field is homogeneous due to diffusion, Eq. (4) with an influx $q_{in} = q_c$ leads to the ideal state at which the volumes of all dunes in the field remain constant. Under this approximation, we consider a simplified model for a dune field that only includes binary collisions.

B. The simplified model

We consider now the simplest approach to a dune field model, namely, a system that consists of a large number of dunes, characterized only by their width, which interact exclusively through collisions between them. We study the evolution of the dune size distribution $P_{col}(w)$ in the entire field in order to check if the macroscopic behavior of the system approaches an absorbent state.

We use two kinds of initializations for the system. In the first the size of the initial dunes (their width) is uniformly distributed between 0 and some w_{max}. The second one generates all dunes with sizes differing only slightly from some previously chosen width w_{char} (strongly peaked distribution). The initial number of dunes is 10^5 in our simulations. During a simulation very small dunes considered unstable are removed from the dune field. In order to simulate an entire dune field, we consider annealed interactions among the dunes. A new mutual lateral distance (offset) is tossed before each collision.

For each collision two dunes are taken randomly from the field to collide. This is repeated every iteration as many times as the number of dunes in the field. No new dunes are generated but

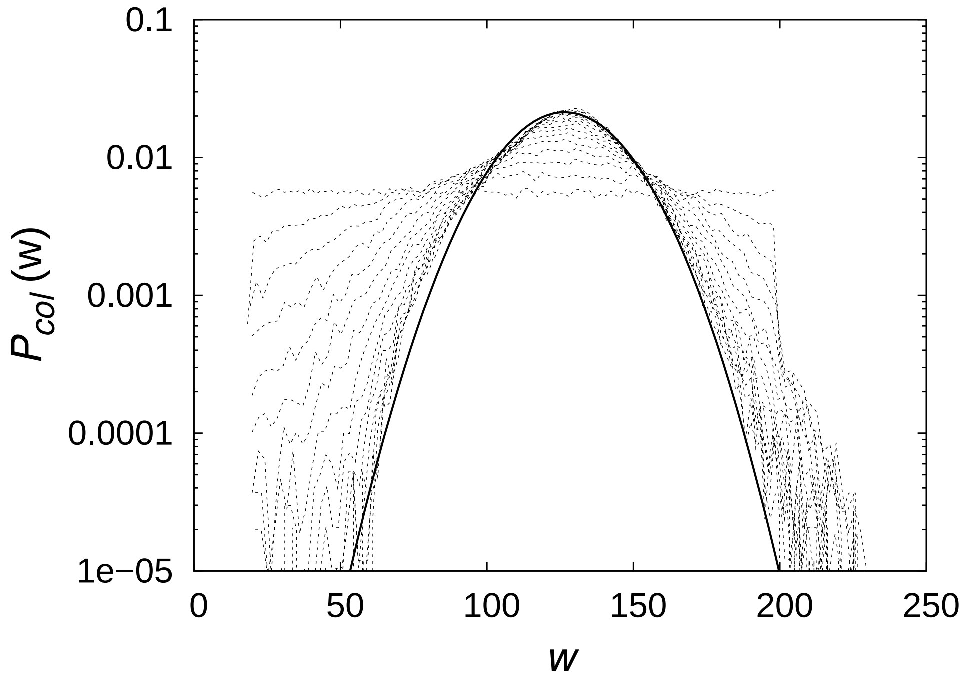

FIG. 13: Snap-shots of the evolution due to binary collisions of a uniform dune size distribution with $w_{max} = 200m$ (dashed-lines) towards a Gaussian (given by a parabola in a semi-log scale, solid line). The final state is reached after about 30 collisions per dune.

the ones with size $w < 1m$ are removed.

The outcome of a collision between the larger dune (width W) and the smaller one (width w) is, for simplicity, assumed to depend only on the relative mutual lateral distance, the random offset θ_i and the volume ratio $r_i = w^3/W^3$, where we use the volume scaling $V \propto w^3$ [3]. The new volume ratio, r_f, is cast, according to the measurements presented above, through Eq. (1), and the conservation of the total volume of the two dunes, $V_{tot} \propto (w^3 + W^3)$. As already mentioned above, Eq. (1) includes only solitary wave behavior and coalescence. We neglect here the creation of new small dunes by breeding. We choose the offset between the two dunes to be a random number distributed uniformly in the interval $[0, 1]$ at each collision.

C. Results

The size distribution function in Fig. 13 shows that dune sizes converge toward an absorbent state with a stable Gaussian–like distribution with mean width $\langle w \rangle_{col}$. The total mass of all dunes is conserved with the exception of a negligible amount due to the small dunes removed from

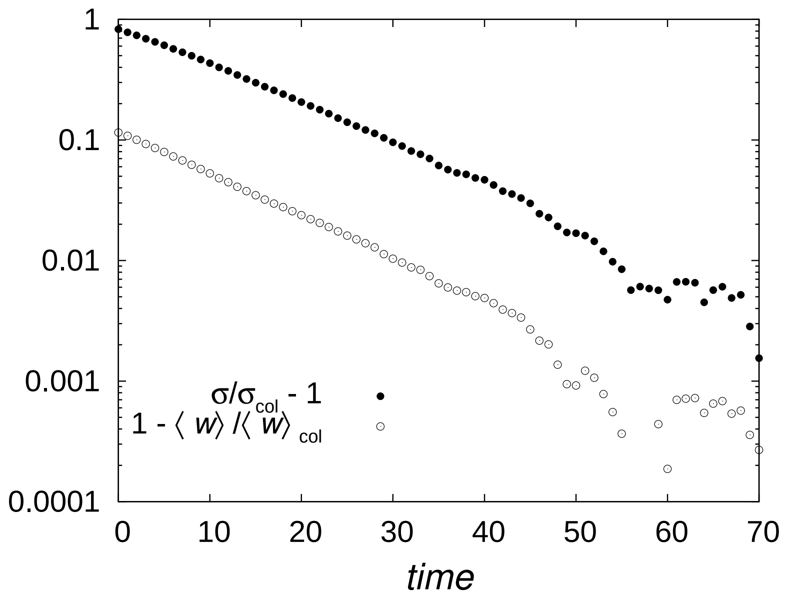

FIG. 14: The first two moments $\langle w \rangle$ (squares) and σ (circles), exponentially relax toward their equilibrium values $\langle w \rangle_{col}$ and σ_{col} (see text). Time units are number of collisions per dune.

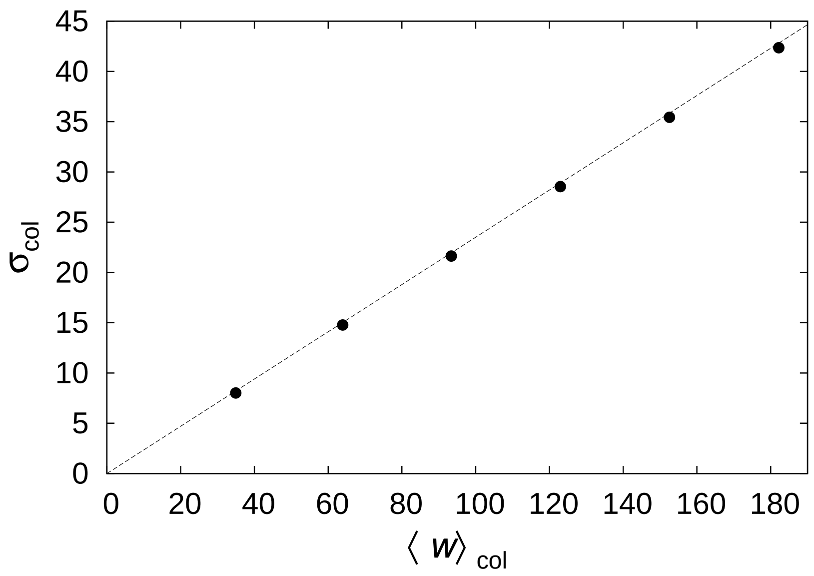

FIG. 15: Linear relation between the two first moments of the observed Gaussian, for different initial volumes of sand available, yielding a constant relative standard deviation $\sigma_{col}/\langle w \rangle_{col}$.

the field. Therefore, the mean dune size $\langle w \rangle_{col}$ is determined by the average volume $\langle V \rangle$. The convergence from the initial distribution to the Gaussian one is found to be a relaxation process with a characteristic time $t_c \sim 10$ *collisions per dune* which arises from the evolution of the

standard deviation and mean size of the actual size distribution showed in Fig. 14.

Furthermore, the mean square deviation σ_{col} is proportional to the mean dune size $\langle w\rangle_{col}$, as shown in Fig. 15. Therefore, the size distribution $P_{col}(w)$ only depends on the average volume of the field $\langle V\rangle$, namely

$$P_{col}(w) = \frac{1}{\sqrt{2\pi}\,\sigma_{col}} \exp -(w - \langle w\rangle_{col})^2/(2\,\sigma_{col}^2). \tag{5}$$

where $\sigma_{col} = a_s\,\langle w\rangle_{col}$, $\langle w\rangle_{col} \approx (\langle V\rangle/c)^{1/3}$ and $a_s \approx 0.24$ results from the fit in Fig. 15. The constant in the volume-size relation: $c \approx 0.017$, is obtained from numerical simulations of isolated barchan dunes under different wind strengths [3].

IV. CONCLUSIONS

The analysis of collisions of barchan dunes with lateral offset showed that some regularity exists in the resulting sizes after the collisions. Some simple relations could be formulated to fit the outcome of such an interaction between two dunes. The resulting volumes can be understood to depend only on the initial offset and volume ratio.

We applied these phenomenological rules to a simple model which considers the effects of collisions in a dune field without considering the sand flux between them. The distribution shows a Gaussian–like shape with respect to the dune sizes. The cubic relation between volume and width of a dune implies that the volume is not Gaussian–distributed as could have been expected.

We showed that dune collisions indeed play a crucial role in selecting a characteristic size and in stabilizing the size distribution of a barchan dune field as already mentioned by ref. [2]. The simple model presented here certainly does not resemble real dune behavior. Especially neglecting the sand flux and the absence of a dependence of the dune interaction on the spatial dune coordinates, hinders us from providing a complete picture of a dune field. The missing mechanisms need to be incorporated to construct a dune field model able to fully understand the effects of collisions onto the macroscopic behavior of a dune field.

Acknowledgments

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