Saturation length

the flux transient defines a spatial lag which is called until the flux gets saturated. The time associated with the number of grains in saltation increases continuously on the sand bed, other grains may be accelerated downwind, impacting after a certain distance back onto the ground. Once a grain collides with the sand bed, other grains may be splashed from the surface, depending on the velocity of the impacting grains [4]. This yields a cascade process in which the number of grains in saltation increases continuously until the flux gets saturated. The time associated with the flux transient defines a spatial lag which is called saturation length \( \ell_s \) and has been shown to scale with

\[
\ell_{\text{drag}} = \frac{\rho_{\text{grain}} d}{\rho_{\text{fluid}}},
\]

where \( d \) is the grain diameter, \( \rho_{\text{grain}} \) is the density of the grain and \( \rho_{\text{fluid}} \) is the density of the driving fluid [5]. The quantity \( \ell_{\text{drag}} \) is the only characteristic length of the saltation dynamics. Recent theoretical studies, supported by previously reported measurements on several terrestrial barchan dune fields, concluded that the minimum dune width \( W_{\text{min}} \) is of the order of 10 times \( \ell_{\text{drag}} \) [6]. On earth, \( \ell_{\text{drag}} \) is around 50 cm, since the atmospheric density is \( \rho_{\text{fluid}} = 1.225 \text{ kg/m}^3 \) and the quartz grains (density \( \rho_{\text{grain}} = 2650 \text{ kg/m}^3 \) of terrestrial dunes have mean diameter \( d = 250 \mu \text{m} \) [3], while the width of the smallest barchan dunes observed is between 5 and 10 m. If we extend this relation to Mars, we expect the smallest dune of basaltic grains (\( \rho_{\text{grain}} = 3200 \text{ kg/m}^3 \) and \( d = 500 \mu \text{m} \) [7]) on Mars to have a width of 1.0 km, since \( \ell_{\text{drag}} \) for the martian scarce atmosphere of density \( \rho_{\text{fluid}} = 0.016 \text{ kg/m}^3 \) in the present is around 100 m. Clearly, the physics behind the above relation between \( \ell_{\text{drag}} \) and the minimum dune size fails for the case Mars, since dunes of only a few hundreds of meters have been observed on Mars. In fact, it has been already noticed [8] that there are differences between martian and terrestrial saltation that must be understood and taken into account to predict the correct minimum dune size.

Under martian atmospheric conditions, the threshold shear velocity \( u_s \) for sustained saltation has been found to be 2.0 m/s, nearly ten times the earth’s [9,10]. It has been shown that when the wind shear velocity \( u_* \) exceeds this threshold value and saltation takes place on Mars, grains travel much faster, have longer trajectories and reach larger heights than their counterparts on Earth [11]. Further, it has been also found that a single grain-bed collision on Mars may result in ejection of hundreds of other grains, whereas on earth only a few grains are splashed after impact [12]. Since the saturation time of the flux is related to the rate grains are carried by the wind into saltation, an estimative of this rate on Mars is fundamental to predict the saturation length and the scale of the dunes.

Recently, a continuum dune model has been introduced [13], which could reproduce quantitatively very well the morphological relations of barchan dunes observed on Earth [14] and solitary wave behavior of dunes [15]. The fundamental idea of the model is to consider the bed-load as a thin fluid-like granular layer on the top of an immobile sand bed. From the shear stress, which is numerically calculated using the algorithm of Hunt et al. [16], the sand flux is computed from a differential equation that contains the saturation transients, and the time evolution of the topography is obtained using mass conservation. If sand deposition leads to slopes larger than the angle of repose \( \theta_r \approx 34^\circ \), avalanches occur and a slip face is formed. In this work, we apply the dune model to simulate dunes in the Arkhangelsky Crater on Mars (41.2°S, 25.0°W). Many of the relevant parameters of the model we need to calculate dunes on Mars are known from the literature, as the mean grain diameter \( d \), the fluid and grain densities, \( \rho_{\text{fluid}} \) and \( \rho_{\text{grain}} \), the threshold shear velocity \( u_* \), for sustained saltation, as well as the angle of repose \( \theta_r \), which is known to be similar as on Earth [9]. The interdune flux \( q_{\text{typ}} \), which is a fraction of the saturated flux \( q_s \), and the shear velocity \( u_* \) are two field variables we want to find from calculations. In the model, the efficiency of the splash is determined by the parameter
\[ \gamma = \frac{dn}{d(\tau_s/\tau_s)}, \]  

which gives the number \( n \) of grains launched into the saltation sheet when the wind strength deviates from the threshold by an amount \( \tau_s/\tau_s \), where \( \tau_s = \rho_{\text{fluid}}(u_s)^2 \) is the threshold wind shear stress, and \( \tau_s = \rho_{\text{fluid}}(u_s)^2 \) is called air borne shear stress, which is lowered if the number of grains in the saltation layer increases and vice versa (“feedback effect”) [13]. The saturation length is calculated as:

\[ \ell_s \propto \frac{l_{\text{drag}}}{\gamma} \left( \frac{u_s}{u_{\text{eff}}} \right)^{\frac{1}{2}} - 1, \]  

and is thus proportional to \( l_{\text{drag}} \) but is also inversely proportional to \( \gamma \) and depends on \( u_s \). While Sauermann et al. [13] have calculated \( \gamma \) on Earth from measurements of the saturation time and found \( \gamma = 0.2 \), we proceed in a different way to find \( \gamma \) on Mars.

After splash, a fraction of the ejected grains enter saltation. If we assume that the physics of the saltation on Mars is the same as on Earth, we expect this fraction, not the total amount, to be of the same order. The parameter \( \gamma \), which gives the rate of grains entering saltation, is proportional to the number of splashed grains \( N \). Since each ejected grain receives a constant fraction of the impact momentum [4], the number of splashed grains is proportional to the ratio between the impact and ejection velocities, \( v_{\text{imp}}/v_{\text{eje}} \), where \( v_{\text{eje}} \) scales with \( \sqrt{gd} \), and \( g \) is the gravity [5]. On the other hand, the impact velocity \( v_{\text{imp}} \) is proportional to the mean grain velocity \( u_s \), which in turn scales with the threshold shear velocity \( u_{\tau_s} \) [13]. Using \( u_{\tau_s} \propto (\rho_{\text{grain}}gd/\rho_{\text{fluid}})^{\frac{1}{2}} \) [3], we obtain \( \gamma \propto u_{\tau_s}/(\sqrt{gd}) \propto (\rho_{\text{grain}}/\rho_{\text{fluid}})^{\frac{1}{2}} \). From this scaling relation, we find that \( \gamma \) on Mars is approximately 2.0, i.e., 10 times the earth’s.

Next, we determine the values of \( u_s/u_{\tau_s} \) and \( q_{\text{in}}/q_s \) for the Arkhangelsky Crater as follows. We found from calculations that the minimal dune width \( W_{\text{min}} \), below which dunes cannot develop neither horns nor slip face and are called domes [5], is proportional to \( \ell_s \). Further, both the constant \( k = W_{\text{min}}/\ell_s \), and the ratio between the minimal dune length \( L_{\text{min}} \) and \( W_{\text{min}} \) depend on \( u_s/u_{\tau_s} \) and \( q_{\text{in}}/q_s \). From the dimensions of the smallest dune in the Arkhangelsky Crater, \( W_{\text{min}} \approx 200 \) m and \( L_{\text{min}} \approx 400 \) m, we find \( q_{\text{in}}/q_s = 0.20 \) and \( u_s/u_{\tau_s} = 1.45 \). This corresponds to \( u_s = 2.8 \) m/s for the Arkhangelsky Crater. Figure 1 shows one image of a dune in the Arkhangelsky Crater together with a calculated dune with similar size. The value \( u_s/u_{\tau_s} = 1.45 \) is similar to the reported ones for terrestrial dune fields [14]. On the other hand, the value of \( \gamma \) (eq. (1)) is 10 times larger on Mars, which we have found to be associated with a larger velocity of the saltating grains on Mars (\( \approx 16.0 \) m/s, whereas \( u_s \) calculated for Earth is nearly 1.6 m/s) and a larger splash.

In conclusion, our calculations show that although \( l_{\text{drag}} \) is the only characteristic length of saltation, the martian splash must be taken into account to predict the saturation length \( \ell_s \) on Mars, which is the relevant length scale of barchan dunes. On the one hand, the atmosphere is 100 times less dense than our planet’s, which increases the saturation length \( \ell_s \). On the other hand, the larger entrainment rate of grains into the saltation layer after grain-bed collisions yields an acceleration of flux saturation and a decrease in the saturation length and the minimal dune width. This property of the martian splash found in the calculations resolves the discrepancy between previously estimated and observed values of minimal dune sizes on Mars.