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ROLLING AND RATCHETING IN SHEAR BANDS

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Abstract.

Using DEM simulations I present results on the internal structure of shear bands obtained under various conditions including two-dimensional Couette cells and uni-axial compression with elastic lateral boundaries. Concerning the velocity profiles and local spin distributions very good quantitative agreement with experiments can be achieved. The spontaneous formation of bearing clusters of finite life-time in systems of round particles is observed. In the case of size distributions following power laws, stable bearings at very high densities can be achieved with Apollonian-like topologies. Such configurations might be relevant for tectonic gouge. In the case of angular particles (convex polygons) local rotations occur in bursts with an energy release distribution that follows a power-law over many orders of magnitude. Also an Omori-like distribution is observed for aftershocks and its exponent depends in the case of anisotropic particles on the initial configuration of the packing. Finally I will also discuss the local restructuring events that are responsible for ratcheting under compression.

One important result of the theory of plasticity is the so-called *shakedown theory.* This theory predicts that a granular material accumulates plastic strains under cyclic loading if the magnitude of the applied load exceeds a threshold value called the shakedown limit. The material is then said to exhibit *incremental collapse* or *ratcheting.* If the loads are below this threshold, the accumulation of permanent deformation stops after a certain number of cycles. However, this basic assumption is difficult to verify by experiments on cyclic loading, because the onset of the ratcheting with the increase of the loading amplitude is gradual and not sharply defined.

Most of the attempts to identify the internal variables of constitutive equations are based on macro-mechanical observations of the response of soil samples in conventional apparatus. A micromechanical investigation would help to select the physically motivated internal variables and to get insight into the principles and mechanics determining their evolution. After all, the mechanical response of granular soils is no more than a combined response of many micromechanical arrangements, such as inter-particle slips, breakage of grains and wearing of the contacts. The development of micromechanical constitutive models is specially motivated by recent experiments on granular materials at grain scale. Using photoelastic disks, these experiments show that stress in granular materials is transmitted through a heterogeneous contact network which reflects a broad contact force distribution. This broadness leads in turn to a considerable number of sliding contacts. These contacts are defined by the condition $|f_t| = \mu f_n$, where f_n and f_t are the normal and tangential contact force and μ the coefficient of friction. Under small deviatoric loads, an initially isotropic packing develops an anisotropic contact network because new contacts are created along the loading direction, while some contacts are lost perpendicular to it. Anisotropy is also observed in the sub-network of sliding contacts, because some contacts leave the sliding condition under slight deviatoric loading. Geometrical anisotropy leads to an anisotropic response of the granular assembly. The effect of the anisotropy of the contact network on the elasto-plastic response has been recently investigated by the introduction of fabric tensors, measuring the orientational distribution of the contacts.

We consider a two dimensional model of convex polygons to represent the grains of the granular material on a mesoscopic scale. The samples consist of isotropic and anisotropic particles in order to study the influence of particle shape anisotropy on the response of a granular packing under very slow shear. The modeling is done by means of the usual discrete element model (DEM), briefly described in this Section. More details and general overview of DEM can be found in [1].

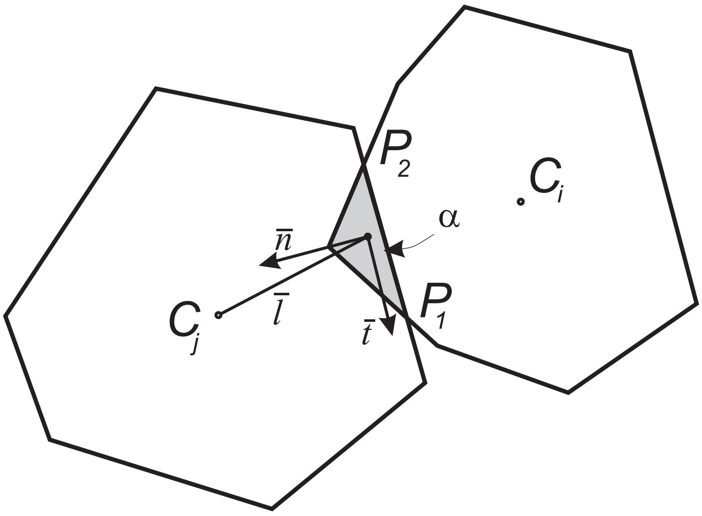

Figure 1: Schematic representation of a particle contact. The overlapping area α is indicated by the shaded zone.

The deformation of the grains is modeled by letting them overlap, as sketched in Fig.1. When two polygons overlap, the intersection points between their edges can be defined.

The segment that connects these points, P_1 and P_2, gives the contact line $S = P_1P_2$. The contact force is given by $f_c = f_e + f_v$, where f_e and f_v are the elastic and damping contribution. The elastic part of the contact force is decomposed as $f_e = f_e\bar{n} + f_e\bar{t}$, where $\bar{n}$ and $\bar{t}$ are the unitary vectors perpendicular and parallel to the contact line S respectively. The normal elastic force is calculated as $f_en = k_n\delta$, where k_n is the normal stiffness, and δ the deformation length defined in terms of the overlapping area a and the length of the contact line, $\delta = a/|S|$. The friction force is given by an elastic force $f_et = k_t\xi$ proportional to the elastic displacement ξ at each contact, with k_t the tangential stiffness. The elastic displacement ξ is updated as $\xi = \xi' + v_c^t\Delta t$, where ξ' is the previous length of the spring, Δt is the time step of the molecular dynamic simulation, and v_c^t the tangential component of the relative velocity v_c at the contact. The length of the tangential spring ξ may increase during the time that the condition $f_t < f_n$ is satisfied. The sliding condition is enforced keeping constant the elastic displacement ξ when the Coulomb limit condition $f_c^t = \mu f_c^n$ is reached and μ is the interparticle friction coefficient. The damping force is calculated as $f_v = -m_v v_c$, where m is the effective mass of the two particles in contact and v_c the damping coefficient. This force takes into account the dissipation at the contact and it is necessary to maintain the numerical stability of the method.

We found that the dynamics of the granular system is characterized by discrete avalanches spanning several orders of magnitude similar to crackling noise[2]. We calculated the probability distribution of the energy released in avalanches, and found it to be in very good agreement with the Gutenberg-Richter law for samples with different particle anisotropy and different system sizes. Consequently the exponent of the released energy distribution can be seen as an invariant property of such systems. Typically this is observed in granular media under very slow shear. For sufficiently large shear rates the results can change qualitatively, namely preventing the occurrence of avalanches. Further, the slow shear rate implicates additional improvements in the numerical integration, as we discuss in previous works.

We also studied the temporal distribution of event sequences after a main-shock. We found that the number of aftershocks decreases with a power of the inverse of time. We could fit the sequences of the rate of aftershocks with the empirical expression and obtained exponents in the range $0.7 < p < 1.6$, similarly to what is observed in real observations according to Omori's law. The anisotropic sample exhibits a larger temporal stability making the temporal occurrence of the avalanches sparser, due to its larger frustration of rotation in the corresponding initial configuration. The larger temporal stability observed at the macro-mechanical level can be therefore taken as an indication of the existence of anisotropic material within the shear zone. This could potentially explain the variation of the exponent p observed in realistic earthquake sequences. The dynamics of the system was also related to the stick-slip process. When one avalanche begins the system slips, while between two successive avalanches, the system sticks, accumulates elastic energy, and becomes weaker because of the increase of the sliding contacts ns. We characterized the weakening of the system by the stiffness and derived the conditional probability P for an avalanche to occur given a fixed stiffness value. We found that P decreases logarithmically with the stiffness and with a decay rate larger for isotropic samples than for anisotropic ones. Concerning frictional strength we found that the probability of an avalanche to occur increases with the force ratio F_s/F_n. The results concerning the conditional probabilities uncovered that anisotropic samples can explore a wider range of stiffness and force ratios than the isotropic sample. This is due to the larger kinematic constraint that anisotropic particles undergo during shear. Further, in general, since the initial configuration corresponding to an anisotropic sample is the most stable configuration with respect to shearing, it is the one with lower probability of failure. The conditional probabilities were computed for a fixed time increment between the observed stiffness and the occurrence of an avalanche and without taken the avalanche magnitude into account. An interesting question for forthcoming studies would be to check if the conditional probability density functions change when only a restricted range of avalanche magnitude values is assumed, but also to see how they depend on the increment time dt which was fixed.

In the quasistatic limit, we have shown the existence of long time regimes with a constant accumulation of plastic deformation per cycle, due to ratchet-like motion at the sliding contacts[3]. As the loading amplitude decreases, a smooth transition from ratcheting to shakedown is observed, which does not allow one to identify a purely elastic regime. The overall response of the polygonal packing under cyclic loading consists of a sequence of long time ratcheting regimes, with slow accumulation of plastic deformation in terms of deviatoric strains, compaction and vorticity. These regimes are separated by short time regimes with large plastic deformations. The analysis of the displacement field per cycle of the particles shows that cyclic loading induce convective motion inside the sample. These motion appears in form of vortex-like structures, which persist during the ratcheting regime.

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