Fiber models for the failure of composite materials

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ABSTRACT

We present recent advances in fiber bundle modeling of the damage and fracture of composite materials focusing on the shear failure of glued interfaces and on the fatigue fracture of bituminous materials. In order to account for complex deformation states arising under shear loading, we discretize the interfaces in terms of beams. Introducing a finite strength of plasticity for broken elements, we capture the load bearing capacity of failed interface regions which remain still in contact during the loading process. To describe fatigue fracture of composites occurring under low amplitude cyclic loading, we propose to enhance fiber bundle models considering a history dependent ageing mechanism for fibers.

1 Introduction

The damage and fracture of composite materials is a very important scientific and technological problem which has attracted an intensive research over the past decades. Composite is a general term here for a broad class of materials which have a strongly heterogeneous microstructure like concrete or asphalt (also called particle composites) and for assembly of subunits which are organized to build up superstructures, e.g. fiber reinforced composites where fiber are embedded in a matrix to improve the mechanical performance of the material. One of the first theoretical approaches to the fracture of composites was the fiber bundle model (FBM) introduced by Peires and Daniels [1, 2]. These early works initiated an intense research in both the engineering and physics communities making fiber models one of the most important theoretical approaches to the damage and fracture of composite materials. Over the past years several extensions of the classical FBM have been worked out considering stress localization, the effect of matrix material between fibers, time dependent response, and thermally activated breakdown [1, 2].

In applications solid bodies are often welded or glued together along an interface which is then expected to sustain various types of external loads. Interfacial failure is the dominating fracture mechanism of fiber reinforced composites under shear loading conditions, caused by the debonding of the fiber-matrix interface. Under shear, interface elements suffer complicated deformations which cannot be represented by fibers but requires a more realistic representation. After debonding, interfaces still can transmit load contributing to the overall load bearing capacity of the system due to, for instance, frictional contacts. Heterogeneous materials subject to periodic loading at an amplitude below their tensile strength can suffer macroscopic failure after a finite number of loading cycles. Such fatigue fracture plays a very important role for the long time performance of particle composites like asphalt in roadways under traffic loading.
In this paper we present recent advances in fiber bundle modeling of the shear failure of glued interfaces [3, 4] and fatigue fracture of bituminous materials like asphalt [5]. In order to account for the complex deformation states of interface elements under shear, we discretize the interface in terms elastic beams which can have stretching and bending deformation and fail due to the two deformation modes [3]. To analyze the effect of the finite load bearing capacity of failed interface regions which remain in contact, we assume that the beams/fibers can have a plastic behavior retaining a fraction of their failure load.

We study in details the effect of the strength of plasticity on the failure process of the system [4]. Finally, we show that fatigue fracture of materials can be studied by means of fiber bundle models by introducing an ageing mechanism of fibers due to the accumulation of damage over the loading history of the system [5].

2 Shear failure of glued interfaces

We propose a novel approach to the shear failure of glued interfaces by extending the classical fiber bundle model to study interfacial failure. Our model represents the interface as an ensemble of parallel beams connecting the surface of two rigid blocks, see Fig. 1. The beams are assumed to have identical geometrical extensions (length l and width d) and linearly elastic behavior characterized by the Young modulus E. In order to capture the failure of the interface, the beams are assumed to break when their deformation exceeds a certain threshold value. Under shear loading of the interface, beams suffer stretching and bending deformation resulting in two modes of breaking. The stretching and bending deformation of beams can be expressed in terms of a single variable, i.e. longitudinal strain $\varepsilon = \Delta l/l$, which enables us to map the interface model to the simpler fiber bundle models. The two breaking modes can be considered to be independent or combined in the form of a von Mises type breaking criterion. The strength of beams is characterized by the two threshold values of stretching $\varepsilon_1$ and bending $\varepsilon_2$ a beam can withstand. The breaking thresholds are assumed to be randomly distributed variables of the joint probability distribution $p(\varepsilon_1, \varepsilon_2)$. Coupling to the rigid blocks ensures that all the beams have the same deformation giving rise to global load sharing, i.e. the load is equally shared by all the elements. Assuming the breaking modes to be independent, a single beam breaks if either its stretching or bending deformation exceeds the respective breaking threshold $\varepsilon_1$ or $\varepsilon_2$, i.e. failure occurs if $f(\varepsilon)/\varepsilon_1 \geq 1$ or $g(\varepsilon)/\varepsilon_2 \geq 1$, where $f(\varepsilon)$ and $g(\varepsilon)$ describe the stretching and bending breaking modes, respectively. Later on this case will be called the OR criterion. The failure functions $f(\varepsilon)$ and $g(\varepsilon)$ can be determined from the elasticity equations of beams; for our specific case of sheared beams they take the form $f(\varepsilon) = \varepsilon$ and $g(\varepsilon) = \sqrt{\varepsilon}$, where $E = 1$ is assumed [3]. In the plane of breaking
Figure 2: a) The plane of breaking thresholds. Fibers which are intact at a deformation $\varepsilon$ fall in the rectangle bounded by $f(\varepsilon), g(\varepsilon)$ and the maximum values of the two thresholds $\varepsilon_1^{\text{max}}, \varepsilon_2^{\text{max}}$ for the OR criterion (area $A + B$), and in the area bounded by the curve connecting $a$ and $b$ and the maximum thresholds for the von Mises type criterion (area $A$), respectively. The fibers which break due to the coupling of the two breaking modes in the von Mises criterion fall in area $B$. b) Comparison of the constitutive curves of a simple fiber bundle and of the beam model with different breaking criteria using uniformly distributed breaking thresholds.

thresholds each beam is represented by a point with coordinates $(\varepsilon_1, \varepsilon_2)$. The constitutive behavior of the interface can be obtained by integrating the load of single beams over the intact ones in the plane of breaking thresholds, see Fig. 2a. For the OR criterion one gets $\sigma = \varepsilon \int_{\varepsilon_1^{\text{min}}}^{\varepsilon_1^{\text{max}}} \int_{\varepsilon_2^{\text{min}}}^{\varepsilon_2^{\text{max}}} d\varepsilon_2 \frac{d\varepsilon_1 p(\varepsilon_1, \varepsilon_2)}{f(\varepsilon)}$. Assuming the thresholds of the two breaking modes to be independently distributed, the disorder distribution factorizes $p(\varepsilon_1, \varepsilon_2) = p_1(\varepsilon_1)p_2(\varepsilon_2)$ and $\sigma(\varepsilon)$ takes the simple form $\sigma(\varepsilon) = \varepsilon [1 - P_1(f(\varepsilon))][1 - P_2(g(\varepsilon))]$. The terms $[1 - P_1(f(\varepsilon))]$ and $[1 - P_2(g(\varepsilon))]$ provide the fraction of beams failed under the stretching and bending breaking modes, respectively. When the two breaking modes are coupled by a von Mises type breaking criterion, a single beam breaks if its strain $\varepsilon$ fulfills the condition $(f(\varepsilon)/\varepsilon_1^{\text{max}})^2 + g(\varepsilon)/\varepsilon_2^{\text{max}} \geq 1$, which is illustrated by Fig. 2a. In this case the constitutive integral $\sigma = \varepsilon \int_{\varepsilon_1^{\text{min}}}^{\varepsilon_1^{\text{max}}} \int_{\varepsilon_2^{\text{min}}}^{\varepsilon_2^{\text{max}}} d\varepsilon_2 \frac{d\varepsilon_1 p(\varepsilon_1, \varepsilon_2)}{f(\varepsilon)}$ cannot be performed explicitly with the integration limit $\varepsilon_2^*(\varepsilon_1, \varepsilon) = \varepsilon_2^0 g(\varepsilon)/(\varepsilon_1^{\text{max}} - f(\varepsilon))$ in general.

In order to determine the behavior of the system for complicated disorder distributions and explore the microscopic failure process of the shear interface, it is necessary to work out a computer simulation technique. For the simulations we consider an ensemble of $N$ beams arranged on a square lattice. Two breaking thresholds are assigned to each beam $i$ ($i = 1, \ldots, N$) of the bundle from the joint probability distribution $p(\varepsilon_1, \varepsilon_2)$. For the OR breaking rule, the failure of a beam is caused either by stretching or bending depending on which one of the conditions is fulfilled at a lower value of the external load. In this way an effective breaking threshold $\varepsilon_i^t$ can be defined for the beams as $\varepsilon_i^t = \min(f^{-1}(\varepsilon_1^t), g^{-1}(\varepsilon_2^t))$, $i = 1, \ldots, N$ where $f^{-1}$ and $g^{-1}$ denote the inverse of $f$, $g$, respectively. For the case of the von Mises type breaking criterion the effective breaking threshold $\varepsilon_i^t$ of beam $i$ can be obtained as the solution of the algebraic equation $(f(\varepsilon_i^t)/\varepsilon_1^{\text{max}})^2 + g(\varepsilon_i^t)/\varepsilon_2^{\text{max}} = 1$. In the case of global load sharing, the load and deformation of beams is everywhere the same along the interface, which implies that beams break in increasing order of their effective breaking thresholds. In the simulation, after determining $\varepsilon_i^t$ for each beam, they
Figure 3: (left) A single fiber shows linearly elastic behavior up to the breaking threshold $\sigma_{th}^i$, then it keeps a fraction $0 \leq \alpha \leq 1$ of the ultimate load $\alpha \sigma_{th}^i$. (right) Macroscopic response of the bundle for a bounded (uniform) $(a,b)$ and unbounded (Weibull) strength distribution $(c,d)$.

are sorted in increasing order. Quasi-static loading of the beam bundle is performed by increasing the external load to break only a single element. Due to the subsequent load redistribution on the intact beams, the failure of a beam may trigger an avalanche of breaking beams. In Fig. 2b the analytic results on the constitutive behavior obtained with uniform distribution of the breaking thresholds are compared to the corresponding results of computer simulations. As a reference, we also plotted the constitutive behaviour of a bundle of fibers where the fibers fail solely due to simple stretching [1, 2]. It can be seen in the figure that the simulation results are in perfect agreement with the analytical predictions. It is important to note that the presence of two breaking modes substantially reduces the critical stress $\sigma_c$ and strain $\varepsilon_c$ ($\sigma_c$ and $\varepsilon_c$ are the value and location of the maximum of the constitutive curves) with respect to the case when failure of elements occurs solely under stretching [1, 2]. The coupling of the two breaking modes in the form of the von Mises criterion gives rise to further reduction of the strength of the interface, see Fig. 2b. Simulations revealed that in spite of the complicated microscopic process of damaging, the size distribution of avalanches shows the same behavior as for simple FBM s, i.e. it has a power law form of an universal exponent 5/2.

3 Bundle of plastic fibers

During the gradual failure of interfaces of solid blocks under shear, damaged regions of the interface can still transmit load contributing to the overall load bearing capacity of the interface. This can occur, for instance, when the two solids remain in contact at the failed regions and exert friction force on each other. In many applications the glue between the two interfaces has disordered properties but its failure characteristics is not perfectly brittle, the glue under shear may also yield carrying a constant load above the yield point. In order to capture this effect in FBM s, we assume that after the breaking of a fiber at the failure threshold $\sigma_{th}^i$, it may retain a fraction $0 \leq \alpha \leq 1$ of its ultimate load $\sigma_{th}^i$, i.e. it will continue to transfer a constant load $\alpha \sigma_{th}^i$ between the surfaces. Plastic behavior implies that the load carried by the broken fibers is independent of the external load, furthermore, it is a random variable due to the randomness of the breaking thresholds (see Fig. 3). Varying the value of $\alpha$, the model interpolates between the perfectly brittle failure ($\alpha = 0$) and perfectly plastic ($\alpha = 1$) behavior of fibers. The load stored by the failed fibers increases the overall strength of the bundle, and it reduces the load
increment redistributed over the intact fibers, which strongly affects the failure process of the interface [4].

Assuming global load sharing (GLS) the macroscopic constitutive equation of the system can be cast in the form

$$\sigma(\varepsilon) = E\varepsilon(1 - P(\varepsilon)) + \alpha \int_0^{\varepsilon} E\varepsilon' p(\varepsilon') d\varepsilon', \quad (2)$$

where the integration is performed over the entire load history. The first term labeled $\sigma_{DFBM}$ provides the load carried by the intact fibers, which corresponds to the classical dry fiber bundle (DFBM) behavior [1, 2]. The second term $\sigma_{pl}$ accounts for the load carried by the broken fibers. It can be seen in Eq. (2) that the value of $\alpha$ controls the relative importance of the elastic and plastic terms influencing the macroscopic response $\sigma(\varepsilon)$ and also the microscopic damage process of the system. The functional form of the constitutive behavior $\sigma(\varepsilon)$ is shown in Fig. 3 for a uniform and a Weibull distribution of disorder. The Weibull distribution has the form $P(\sigma_{th}) = 1 - \exp\left(-\frac{\sigma_{th}}{\lambda}\right)^m$, where $\lambda$ sets the characteristic strength and $m$ controls the amount of disorder. It is interesting to note that for $\alpha < 1$ there always exists a maximum of $\sigma(\varepsilon)$, just as in the case of DFBM [1, 2]. Under stress controlled loading conditions, macroscopic failure occurs at the maximum of $\sigma(\varepsilon)$ so that the position and value of the maximum define the critical stress $\sigma_c$ and strain $\varepsilon_c$ of the bundle, respectively. It can be observed in Fig. 3 that the value of $\sigma_c$ and $\varepsilon_c$ are both higher than the corresponding values of DFBM indicating that the presence of plastic fibers increases the macroscopic strength of the bundle. For the limiting case $\alpha \to 1$ the maximum of $\sigma(\varepsilon)$ gradually disappears and the macroscopic response of the bundle becomes perfectly plastic.

From experimental and theoretical point of view, it is very important to study the behavior of the plastic bundle when the interaction of fibers is localized. In the case of local load sharing (LLS) under stress controlled external loading conditions, the load dropped by the broken fiber is redistributed in a local neighborhood of the fiber giving rise to high stress concentration in the vicinity of failed regions. Stress concentration leads to correlated growth of clusters of broken fibers (cracks), which plays a crucial role in the final breakdown of the system, i.e. macroscopic failure of the bundle occurs due to the
instability of a broken cluster which then triggers an avalanche of failure events where all the remaining intact fibers break. This effect typically leads to a more brittle constitutive behavior of the system and the appearance of non-trivial spatial and temporal correlations in the damage process. We simulated the fracture process of a bundle of $N$ plastic fibers organized on a square lattice, redistributing the load over the four nearest neighbors of the broken elements. Computer simulations showed that the presence of load bearing broken fibers has a substantial effect on the failure process of the bundle. Increasing the value of $\alpha$ the amount of load $(1 - \alpha)\sigma_{th}$ redistributed locally is decreased reducing the stress concentration around failed regions. It has the consequence that there exists a critical value $\alpha_c$ above which locally correlated growth of cracks due to stress concentrations becomes negligible and the macroscopic response of the LLS bundle becomes identical to its GLS counterpart. This is illustrated in Fig. 4 where the difference of the critical stresses $\sigma^{GLS}_c$ and $\sigma^{LLS}_c$ of the GLS and LLS bundles is presented as a function of $\alpha$ for Weibull distributions with different Weibull exponents $m$. It can be seen that a well defined critical value $\alpha_c$ can be identified, above which the difference takes practically zero value [4].

The microscopic process of failure is characterized by the spatial structure of clusters of broken fibers and by the temporal correlations of local breakings (avalanches of breaking events). Computer simulations revealed that the size distribution of bursts $D(\Delta)$ of simultaneously failing fibers becomes a power law at the critical point $\alpha_c$ with an exponent $\mu = 1.5$ (see Fig. 4). When the load sharing is localized it is found that the load carried by the broken fibers has a stabilizing effect on the bundle, i.e. it lowers the stress concentration around clusters of failed fibers. Consequently, the clusters get larger with increasing $\alpha$ as it is illustrated in Fig. 5. The quantitative characterization of the cluster structure is provided by the distribution $n(S)$ of cluster sizes $S$, which again becomes a pure power law $n(S) \sim S^{-\tau}$ at the critical value of the control parameter $\alpha_c$ with an exponent $\tau = 2.35 \pm 0.08$ (see Fig. 5).

The failure process of the bundle is dominated by the competition of fiber breaking by local stress enhancement due to load redistribution and by local weakness due to disorder. Our detailed analysis revealed that the relative importance of the two effects is
controlled by the parameter $\alpha$. Below the critical point $\alpha < \alpha_c$ high stress concentration can develop around cracks so that the failure of the bundle occurs due to localization. Above the critical point $\alpha \geq \alpha_c$ the macroscopic response of the LLS bundle becomes practically identical with the GLS constitutive behavior showing the dominance of disorder. Analyzing the evolution of the micro-structure of damage with increasing $\alpha$, the transition proved to be continuous analogous to percolation [4].

4 Fiber bundle model for fatigue failure

Materials subject to periodic external loading with an amplitude below the tensile strength, often show a gradual accumulation of deformation which can even lead to macroscopic failure over a finite time. This subcritical crack growth and failure called fatigue fracture is one of the most important processes which limits the lifetime of structural components in applications. Fatigue fracture is the typical distress of asphalt in pavements and roadways due to the repeated traffic loading. From structural point of view asphalt is a particle composite where aggregates (crushed stones) are embedded in a polymer matrix (asphalt binder). To obtain a quantitative characterization of the fatigue fracture of asphalt, we carried out experiments studying the cyclic compression of cylindrical asphalt specimens at a constant amplitude $\sigma_0$ below the tensile strength $\sigma_c$ of the material. Figure 6a shows that increasing the number of loading cycles $N_{cycle}$ the deformation $\varepsilon$ accumulates and the system approaches macroscopic failure at a finite number $N_f$ of cycles with a diverging deformation rate $d\varepsilon/dt$. Studying the lifetime $N_f$ of the specimen as a function of the external load $\sigma_0/\sigma_c$ (Fig. 6b) three regimes of the fatigue process can be distinguished: approaching the tensile strength $\sigma_0 \rightarrow \sigma_c$ rapid failure occurs, while at the other extreme a so-called fatigue limit $\sigma_f/\sigma_c$ can be identified below which no macroscopic failure occurs and the system has an infinite lifetime. For intermediate load values a so-called Basquin regime is found, where the lifetime has a power law dependence of the external load $N_f \sim (\sigma_0/\sigma_c)^{-\alpha}$. For the exponent $\alpha = 2.2 \pm 0.1$ was obtained (Fig. 6b). Experiments have revealed that asphalt subject to repeated loading at low amplitude typically fails due to three microscopic mechanisms: damage accumulation, healing of existing microcracks and stress relaxation due to viscoelasticity. In spite of the large amount of experimental results gathered over the past decades, the comprehensive theoretical understanding of the process is still lacking.
Recently, we introduced a fiber bundle model which captures the main ingredients of the fatigue failure of asphalt [5]. The model discretizes the thin layer of the asphalt matrix where fatigue crack growth occurs in terms of a parallel bundle of fibers with identical Young modulus $E$. Fibers fail due to two physical mechanisms, namely, immediate breaking occurs when the local load exceeds the strength of fibers; furthermore, intact fibers undergo a damage accumulation process by the nucleation of microcracks during the loading history of the system. Fibers are assumed to have a finite damage tolerance, i.e., when the amount of accumulated damage reaches a threshold value the fiber fails. The two breaking thresholds of immediate breaking and damage tolerance are independent random variables. Healing of microcracks is captured in the model by limiting the range of memory of the system over which the loading history contributes to the accumulated damage. Assuming global load sharing, under a constant external load $\sigma_0$ the evolution equation of the fiber bundle for fatigue failure can be cast in the form

$$\sigma_0 = [1 - F(a \int_0^t e^{-\frac{\{(a+\beta)\}}{\gamma} p(t')\gamma dt'})] [1 - G(p(t))] p(t),$$

where $F$ and $G$ are the cumulative distributions of the threshold values of damage tolerance and immediate breaking, respectively. Eq. (3) has to be solved for the load of single fibers $p(t)$ as a function of time $t$, which is simply related to the deformation of the bundle $p(t) = E\varepsilon(t)$. The exponential term in the argument of $F$ takes into account the healing of microcracks by limiting the range of memory to a finite value $\tau$. Figure 6a, b) demonstrate that our model provides an excellent quantitative agreements with the experimental findings.

5 Concluding remarks

We have demonstrated that the classical FBM can be improved to account for complex deformation states, plastic behavior and ageing of materials providing a quantitative insight into the failure process of a broad class of composite materials. In the limiting case of global load sharing most of the characteristic quantities can be obtained in closed analytic forms, while the realistic treatment of localized interactions requires large scale computer simulations.

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References


