Fragmentation

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Brittle materials fragment when exploded or under impact. The study of fragmentation is of practical importance in many areas, ranging from archaeology to milling. In the last ten years much progress has been achieved in the understanding of the fragment size and velocity distributions as function of the total energy, the geometry and the material strength. Scaling laws, analogous to those of critical phenomena, have been formulated. Recent experiments of exploding egg shells and christmas balls have given insight also into the fragmentation of containers. For the case of shells, new critical exponents are obtained. These results are confirmed by numerical simulations. These laws are important to understand space debris.

I. INTRODUCTION

Fragmentation, *i.e.* the breaking of particulate materials into smaller pieces is abundant in nature and underlies several industrial processes, which attracted a continuous interest in scientific and engineering research over the past decades [1–16]. Fragmentation phenomena can be observed on a broad range of length scales ranging from the collisional evolution of asteroids and meteor impacts on the astrophysical scale, through geological phenomena and industrial applications on the intermediate scale down to the break-up of large molecules and heavy nuclei on the atomic scale. In laboratory experiments on the fragmentation of solids, the energy input is usually achieved by shooting a projectile into a solid block [4–9], making an explosion inside the sample [2, 3] or by the collision of macroscopic bodies (free fall impact) [11–16]. Due to the violent nature of the process, observations on fragmenting systems are often restricted to the final state, making the fragment size (volume, mass, charge, ...) to be the main characteristic quantity. The most striking observation on fragmentation is that the distribution of fragment sizes shows a power law behavior, independently on the way of imparting energy, relevant microscopic interactions and length scales involved, with an exponent depending only on the dimensionality of the system [2–17]. During the past years experimental [2–17] and theoretical [18–41] efforts focused on the validity region and the reason of the observed universality in 1, 2, and 3 dimensions. Detailed studies revealed that universality prevails for large enough input energies when the system falls apart into small enough pieces [7–10, 13–16, 18–27], however, at lower energies a systematic dependence of the exponent on the input energy was evidenced [28, 29]. Recent investigations on the low energy limit of fragmentation suggest that the power law distribution of fragment sizes arises due to an underlying critical point [21, 22, 27, 34, 35, 38].

Besides the industrial and social impact of the failure of shell like systems, they are also of high scientific importance for the understanding of fragmentation phenomena. Former studies on fragmentation have focused on the behavior of bulk systems in one, two and three dimensions under impact and explosive loading, however, hardly any studies have been devoted to fragmentation of shells [38]. The peculiarity of the break-up of closed shells originates from the fact that the local structure is inherently two-dimensional, however, the dynamics of the systems, the motion of material elements, deformation and stress states are three-dimensional which allows for a rich variety of failure modes [38].

Closed shells made of solid materials are often used in every day life, industrial applications and engineering practice as containers, pressure vessels or combustion chambers. From a structural point of view aircraft vehicles, launch vehicles like rockets and building blocks of a space station are also shell-like systems, and even certain types of modern buildings can be considered as shells. The egg-shell as nature's oldest container proved to be a reliable construction for protecting life. In most of the applications shell-like constructions operate under an internal pressure much higher than the surrounding one. Hence, careful design and optimization of structural and material properties is required to ensure the stability and reliability of the system. Closed shells usually fail due to an excess internal load which can arise either as a result of slowly driving the system above its stability limit during its usage or service time, or by a pressure pulse caused by an explosive shock inside the shell. Due to the widespread applications, the failure of shell systems is a very important scientific and technological problem which has also an enormous social impact due to the human costs arising, for instance, in accidental events.

II. SOME EXPERIMENTS

Hen eggs provide an excellent possibility for the study of fragmentation of thin brittle shells of disordered materials with the additional advantages of being cheap and easy to handle, making the patience of scientists the only limiting factor for the subsequent improvement of the ex-

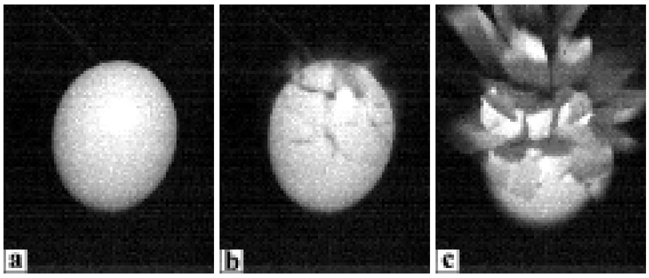

FIG. 1: Time evolution of the explosion of an egg-shell, consecutive snapshots taken by a high speed camera. The time difference between the snapshots is 0.001 sec.

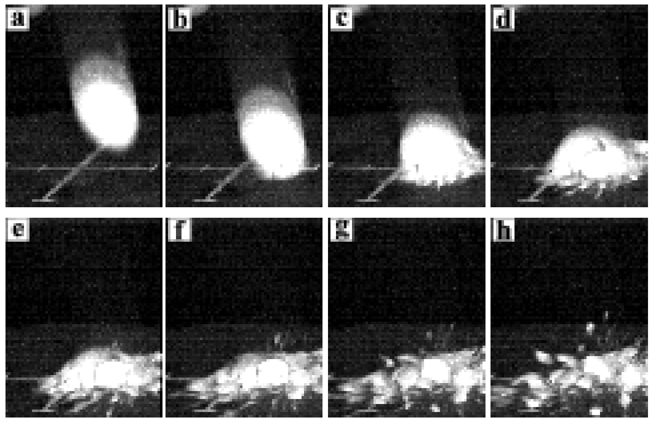

FIG. 2: Time series of the impact of an egg-shell with the hard ground. The consecutive snapshots were taken by a high speed camera of 1 kHz.

perimental results. Our experiments were performed on ordinary brown and white egg-shells. In the preparations, first two holes of regular circular shape were drilled on the bottom and top of the egg through which the content of the egg was blown-out. The inside was carefully washed and rinsed out several times and finally the empty shells were dried in a microwave oven to get rid of all moisture of the egg-shell. In the impact experiments intact egg-shells are catapulted onto the ground at a high speed using a simple setup of rubber bands. The experimental setup provided a relatively high energy impact without the possibility of varying the imparted energy. The eggs are shot directly into a plastic bag touching the ground so that no fragments are lost for further evaluation.

In the explosion experiment initially the egg-shell is flooded with hydrogen and hung vertically inside a plastic bag. The combustion reaction is initiated by igniting the escaping hydrogen on the top of the egg. The hydrogen immediately reacts with the Oxygen which is also drawn up into the egg through the bottom hole, mixing with the remaining hydrogen. When enough air has entered to form a combustible mixture inside the egg, the flame back-fires through the top hole and starts the very quick exothermic reaction. The experiment is carried out inside a soft plastic bag so that secondary fragmentations due to fragment-wall collisions do not occur, furthermore, no pieces were lost after explosion. Since the pressure which builds up during combustion can slightly be changed by the hole size, *i.e* the smaller the hole, the higher the pressure at the explosion, we performed several series of experiments with hole diameters d between 1.2 and 2.5 millimeter.

It is possible to follow the time evolution of the explosion and impact processes by means of a high speed camera under well controlled conditions. Three consecutive snapshots of the explosion process are presented in Fig. 1 taken by a camera of 1000 Hz frequency. The ignition took place at the top of the egg in Fig. 1a). The instant of back-firing and the initiation of combustion is captured in Fig. 1b), while in Fig. 1c) already the flying pieces can be seen. Based on the snapshots the total duration of an explosion is estimated to be of the order of 1 millisecond.

In the impact experiment the egg hits the ground in the direction of its longer axis, as it is illustrated by the picture series of Fig. 2. After hitting the ground (Fig. 2b), the egg suffers gradual collapse as it moves forward (Fig. $2c-h$) making the impact process relatively longer compared to the explosion.

The resulted egg-shell pieces are then carefully collected and placed on the tray of a scanner without overlap. In the scanned image fragments are seen as black spots on a white background and were further analyzed by a cluster searching code. In the inset of Fig. 3 an example of scanned pieces of an impact experiment is shown where the broad variation of sizes can also be noticed with the naked eye. A dusty phase of shattered pieces [42] was also observed in the experiments with fragment sizes falling in the order of the pixel size of the scanner.

As the main quantitative result of the experiments we evaluated the mass distribution of fragments $F(m)$ which is defined so that $F(m)\Delta m$ provides the probability of finding a fragment with mass falling between m and $m+\Delta m$. Fig. 3 presents the fragment mass distributions $F(m)$ for impact and explosion experiments averaged over 10-20 egg-shells for each curve. For the impact experiment, a power law behavior of the fragment mass distribution

$$F(m) \sim m^{-\tau} \qquad (1)$$

can be observed over three orders of magnitude where the value of the exponent can be determined with high precision to $\tau = 1.35 \pm 0.02$. Explosion experiments result also in a power law distribution of the same value of τ for small fragments with a relatively broad cut-off for the large ones. Smaller hole diameter d in Fig. 3, *i.e.* higher pressure, gives rise to a larger number of fragments with a smaller cut-off mass and a faster decay of the distribution $F(m)$ at the large fragments. Note that the relatively small value of the exponent τ can indicate a cleavage mechanism of shell fragmentation and is signifi-

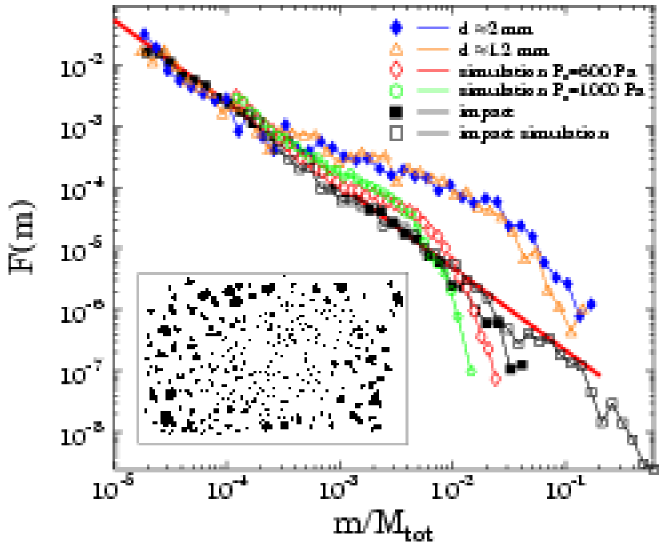

FIG. 3: Comparison of fragment mass distributions obtained by explosion experiments with two hole sizes and the impact experiment to the simulation results. The inset shows a typical scanned set of fragments.

cantly different from the experimental and theoretical results on fragmenting two-dimensional bulk systems where $1.5 \leq \tau \leq 2$ has been found [2, 7, 8, 13–16, 21, 22, 34–37], and from the three-dimensional ones where $\tau > 2$ is obtained [2, 5, 6, 39, 40].

III. SIMULATIONS

Most of the theoretical studies on fragmentation relay on large scale computer simulations since capabilities of analytic approaches are rather limited in this field due to the complexity of the break-up process. Over the past years the Discrete Element Method (DEM) proved to be a very efficient numerical technique for fragmentation phenomena [21, 22, 27, 34–41].

In order to investigate the fragmentation of spherical shells we constructed a three-dimensional discrete element model such that the surface of the unit sphere is discretized into randomly shaped triangles (Delaunay triangulation) by throwing points randomly and independently on the surface [43, 44]. The nodes of the triangulation represent point-like material elements in the model whose mass is defined by the area of the Voronoi polygon assigned to it [36, 43, 44]. The bonds between nodes are assumed to be springs having linear elastic behavior up to failure. Disorder is introduced in the model solely by the randomness of the tessellation so that the mass assigned to the nodes, the length and cross-section of the springs are determined by the tessellation (quenched structural disorder). After prescribing the initial conditions of a specific fragmentation process, the time evolution of the system is followed by solving the equation of motion of nodes by a Predictor-Corrector method of fourth order

$$m_i \ddot{\vec{r}}_i = \vec{F}_i^s + \vec{F}_i^{ext} + \vec{F}_i^d, \quad i = 1, \ldots N, \tag{2}$$

where $\vec{F}_i^s$ is the sum of forces exerted by the springs connected to node i, and $\vec{F}_i^{ext}$ denotes the external driving force, which depends on the loading condition. To facilitate the relaxation of the system at the end of the fragmentation process, a small viscous damping force $\vec{F}_i^d$ was also introduced in Eq. (2).

In order to account for crack formation in the model springs are assumed to break when their deformation ε exceeds a certain breaking threshold ε_c. A fixed threshold value $\varepsilon_c = 0.03$ is set for all the springs resulting in a random sequence of breakings due to the disordered spring properties. The breaking criterion is evaluated at each iteration step and those springs which fulfill the condition are removed from the simulation. As a result of successive spring breakings cracks nucleate, grow and merge on the spherical surface which can give rise to a complete break-up of the shell into smaller pieces.

Fragments of the shell are defined in the model as sets of nodes (material elements) connected by the remaining intact springs. The process is stopped when the system has attained a relaxed state, *i.e.* when there is no spring breaking over a large number of iteration steps.

In computer simulations two different ways of loading have been considered which model the experimental conditions and represent limiting cases of energy input rates: (i) *pressure pulse* and (ii) *impact* load. A pressure pulse in a shell is carried out by imposing a fixed internal pressure P_o from which the forces $\vec{F}_j^{ext}$ acting on the triangular surface elements are calculated as

$$\vec{F}_j^{ext} = P_o A_j \vec{n}_j, \tag{3}$$

where A_j denotes the actual area of triangle j and the force points in the direction of the local normal $\vec{n}_j$. The force $\vec{F}_j^{ext}$ is equally shared by the three nodes of the triangle for which the equation of motion Eq. (2) is solved. The impact loading realizes the limiting case of instantaneous energy input by giving a fixed initial radially oriented velocity v_o to the material elements and following the resulted time evolution of the system by solving the equation of motion Eq. (2).

IV. FRAGMENT MASSES

Large scale simulations have been performed varying the control parameters, *i.e.* the fixed pressure P_o, and the imparted energy E_o over a broad range. In the simulations two cut-offs arise for the fragment masses, where the lower one is defined by the single unbreakable material elements of the model and the upper one is due to the finite size of the system.

The most important characteristic quantity of our system which can also be compared to the experimental results is the mass distribution of fragments $F(m)$. Under impact loading for $E_o < E_c$ we found that $F(m)$ has a pronounced peak at large fragments indicating the presence of large damaged pieces. Approaching the critical point E_c the peak gradually disappears and the distribution asymptotically becomes a power law at E_c. Above the critical point the power law remains for small fragments followed by a cut-off for the large ones, which decreases with increasing E_o.

For pressure loading $F(m)$ can only be evaluated above P_c and it always shows a power law behavior for small fragments with a relatively broad cut-off for the large ones. For the purpose of comparison, a mass distribution $F(m)$ obtained at an impact energy close to the critical point E_c, and distributions at two different pressure values P_o of the ratio 1.6 are plotted in Fig. 3 along with the experimental results. For impact an excellent agreement with the experimental and theoretical results is evidenced. For pressure loading, the functional form of $F(m)$ has a nice qualitative agreement with the experimental findings on the explosion of eggs, furthermore, distributions at the same ratio of pressure values obtained by simulations and experiments show the same tendency of evolution, see Fig. 3.

Fig. 4 demonstrates that by rescaling the mass distributions above the critical point by plotting $F(m) \cdot \overline{M}^{\delta}$ as a function of $m/\overline{M}$ an excellent data collapse is obtained with $\delta = 1.6 \pm 0.03$. The data collapse implies the validity of the scaling form

$$F(m) \sim m^{-\tau} \cdot f(m/\overline{M}), \quad (4)$$

typical for critical phenomena. The cut-off function f has a simple exponential form $\exp\left(-m/\overline{M}\right)$ for impact loading, and a more complex one containing also an exponential component for the pressure case (see Fig. 4). The average fragment mass $\overline{M}$ occurring in the scaling form Eq. (4) diverges according to a power law when approaching the critical point. The good quality of collapse and the functional form Eq. (4) also imply that the exponent τ of the mass distribution does not depend on the value of the pressure P_o or the kinetic energy E_o contrary to the bulk fragmentation where an energy dependence of τ was reported [28].

The rescaled plots make possible an accurate determination of the exponent τ, where $\tau = 1.35 \pm 0.03$ and $\tau = 1.55 \pm 0.03$ were obtained for impact and pressure loading, respectively. Hence, a good quantitative agreement of the theoretical and experimental values of the exponent τ is evidenced for the impact loading of shells, however, for the case of pressure loading the numerically obtained exponent turned out to be somewhat higher than in the case of exploded eggs.

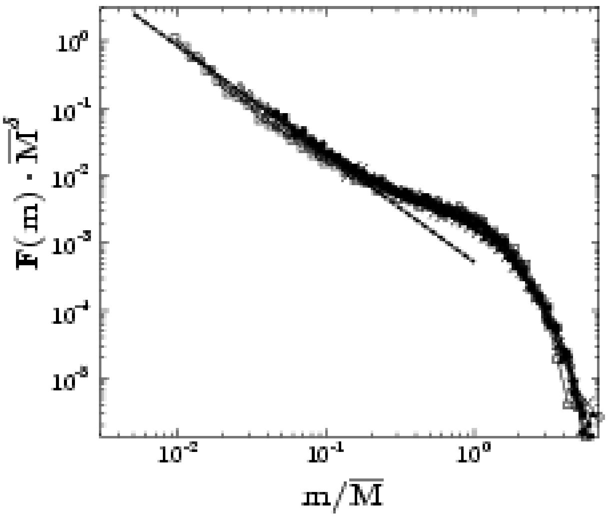

FIG. 4: Rescaled plot of the mass distributions of various pressure values above the critical point $P_o > P_c$. The dashed line indicates the fitted power law with an exponent $\tau = 1.55 \pm 0.03$.

V. DISCUSSION AND OUTLOOK

We presented a detailed experimental and theoretical study of the break-up of closed shells arising due to a shock inside the shell. For the purpose of experiments brown and white hen egg-shells were carefully prepared to ensure a high degree of brittleness of the disordered shell material. The break-up of the shell was studied under two different loading conditions, *i.e.* explosion caused by a combustible mixture and impact with the hard ground. As the main outcome of the experiments, the mass distribution of fragments proved to be a power law in both loading cases for small fragment sizes, however, qualitative differences were obtained in the limit of large fragments for the shape of the cut-off.

Simulations revealed that depending on the value of P_o and E_o, the final outcome of the break-up process can be classified into two states, *i.e.* damaged and fragmented with a sharp transition in between at a critical value of the control parameters P_c and E_c. In the fragmented regime power law fragment mass distributions were obtained in satisfactory agreement with the experimental findings. Analyzing the behavior of the system in the vicinity of the critical point P_c, E_c, we showed that power law distributions arise in the break-up of shells due to an underlying phase transition between the damaged and fragmented states, which proved to be abrupt for explosion, and continuous for impact.

Due to its unique characteristics, the break-up of shells defines a new universality class of fragmentation phenomena, different from that of the two- and three-dimensional bulk systems. Based on universality, our results should be applicable to describe the break-up of other closed shell systems composed of disordered brittle materials. Explosion of shell-like fuel containers, tanks, high pressure vessels often occur as accidental events in industry, or in space missions where also the explosion of complete satellites may occur creating a high amount of space debris orbiting about Earth. For the safety design of shell constructions, and for the tracking of space debris it is crucial to have a comprehensive understanding of the break-up of shells. For that purpose a very useful dicovery was, that depending on the material the surface to volume ratio can scale differently giving for instance for glass self-affine fragment shapes [45].

Acknowledgments

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