

Numerical methods for aeolian transport

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Abstract:

The motion of sand driven by the wind is one of the most common geomorphological phenomena. It is responsible for the creation of dunes and the covering by sand of vast areas, in particular in the arid regions. Its understanding and numerical reproduction is of big practical importance since the economic losses due to the mobility of sand are huge in many places. Until a few years ago no consistent equation of motion for aeolian driven surfaces was available. I will report on the recent progress in this respect. A system of three coupled equations describing the shear stress of the wind on the ground, the flux of sand on the surface and the topography are derived using the basic transport mechanism of saltation. These equations are validated on velocity, flux and shape measurements performed on desert and coastal dunes in Morocco and Brazil. Then the equations are used to simulate the dynamics of dunes and show among others solitary wave behaviour of so-called Barchan dunes and the stability of dune fields.

Keywords: granular media, sand transport, fluid dynamics

1. Introduction

If granular materials are submerged into a fluid like air or water which is in motion, this fluid will exert forces on the grains and in that way create a particle flux. The sand flux on the surface modifies the shape of the landscape and spontaneously creates patterns on different scales: ripples in the range of ten to twenty centimeters and dunes in the range of two to two hundred meters. This mobile topography can be described by a set of coupled equations of motion which contain as variable fields the wind shear stress and the sand flux.

2. The wind shear stress

A dune or a smooth hill can be considered as a perturbation of the surface that causes a perturbation of the air flow. An analytical calculation of the shear stress perturbation due to a two dimensional hill has been performed first by Jackson and Hunt [1]. Later, the work has been extended to three dimensional hills and further refined [2, 3]. The following discussion is mainly based on the work of ref. [3].

They obtain after a lengthy calculation for the Fourier transformation of the shear stress perturbation $\hat{\tau}_x$ in wind direction,

$$\hat{\tau}_x(k_x, k_y) = \frac{h(k_x, k_y)k_x^2}{|k|} \frac{2}{v^2(l)} \cdot \left\{ 1 + \left(2 + \ln \frac{l}{z_0} \frac{|k|^2}{k_x^2} \right) \frac{\sigma}{\ln l/z_0} \frac{K_1(2\sigma)}{K_0(2\sigma)} \right\}, \quad (1)$$

and the shear stress perturbation $\hat{\tau}_y$ in lateral direction,

$$\hat{\tau}_y(k_x, k_y) = \frac{h(k_x, k_y)k_x k_y}{|k|} \frac{2}{v^2(l)} 2\sqrt{2} \sigma K_1(2\sqrt{2} \sigma), \quad (2)$$

where $|k| = \sqrt{k_x^2 + k_y^2}$, K_0 and K_1 are modified Bessel functions and $\sigma = \sqrt{i L k_x z_0/l}$. L is the typical length scale of the dune given by one quarter of the mean wave length obtained from the Fourier representation of the shape. Furthermore, $v(l)$ denotes the dimensionless velocity at the height l normalized by the velocity v_0 ,

$$v(l) = \frac{u_*}{v_0 \kappa} \ln \frac{l}{z_0}, \quad (3)$$

where $\kappa \approx 0.4$ is the von Kármán's constant and v_0 the velocity of the undisturbed upwind profile at the intermediate height. While the Bessel functions are computationally expensive to evaluate, a suitable approximation can be found which is both sufficiently accurate and efficient. Figure 1 skews the shear stress over a cosine-shaped hill.

3. Aeolian sand transport

Analytical calculations that predict the sand flux by averaging over the microscopic processes have deepened the understanding of aeolian sediment transport very much. A result of such a theoretical calculation was obtained by Sørensen [4],

$$q_S = C_S \frac{\rho_{\text{air}}}{g} u_* (u_* - u_{*t}) (u_* + 7.6 * u_{*t} + 2.05 \text{ ms}^{-1}), \quad (4)$$

where C_S is a parameter that has analytically been determined. Fitting eq. (4) to wind tunnel data revealed that the

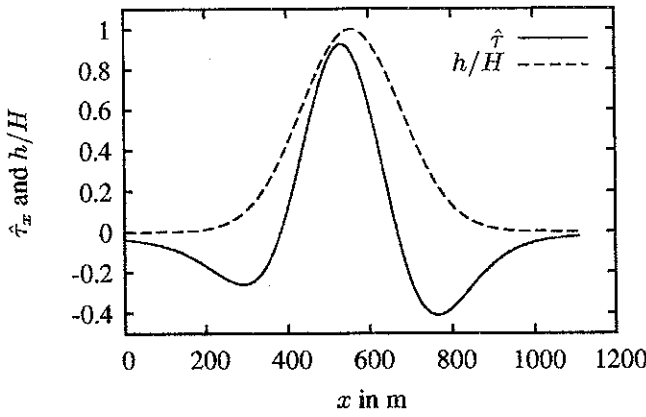


Figure 1: Shear stress at the surface calculated according to eqn. (1). The maximum of the shear stress is shifted upwind with respect to the maximum of the profile $h(x)$.

analytical value of C_S is about four times too small, however, the functional structure of the relation reproduces the data very well [5].

All the preceding relations of the form $q(u_*, \dots)$ assume that the sediment transport is in steady state, i.e. the sand flux is saturated. In order to overcome this limitation and to get information about the dynamics of the aeolian sand transport, numerical simulations based on the grain scale have been performed [6–8]. They showed that on a flat surface the typical time to reach the equilibrium state in saltation is approximately two seconds, which was later confirmed by wind tunnel measurements [9].

If one assumes that each splash event produces on average the same number of ejected new particles the increase of saltating grains would be exponential in time. After a saturation time T_s , however, the flux saturates to q_s . From this microscopic picture Sauermann et al. [10, 11] have derived the equation for the evolution of the flux

$$\frac{\partial q}{\partial x} = \frac{1}{l_s} q \left(1 - \frac{q}{q_s} \right), \quad (5)$$

where l_s is the saturation length.

4. Dunes

The full three dimensional dune model is defined by the three variable fields $h(x, y)$, $q(x, y)$ and $\tau(x, y)$. τ is calculated from h through the Fourier-space equations (1) and (2). Then q is obtained from τ through eq. (5). Then the new topography h is obtained from q using mass conservation:

$$\frac{\partial h}{\partial t} = \frac{1}{\rho_{\text{sand}}} \nabla_s q \quad (6)$$

If $\nabla_s h > \tan \Theta$, where Θ is the angle of repose, avalanche relaxation equations are applied. ∇_s denotes the spatial derivative in direction of the strongest gradient in wind direction. Once h is obtained one goes back to calculate again τ etc. With this loop one iteratively solves the time evolution of the three fields.

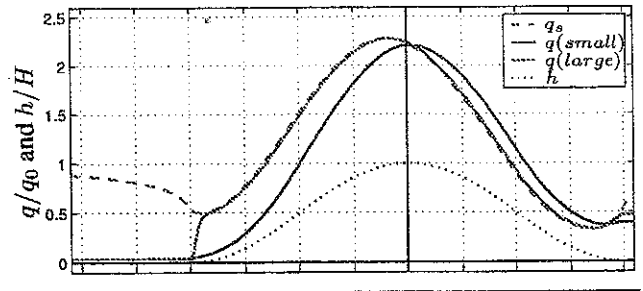


Figure 2: The figure shows the saturated flux q_s , eq. (4), and the flux q , eq. (5), including saturation transients, calculated for two cosine shaped hills with an aspect ratio of $H/L = 1/8$. The hills have a height of $H = 1$ m and $H = 10$ m, the saturation length is $l_s = 0.8$ m. The saturated flux q_s is scale invariant and thus identical for both hills, but the flux q gets completely different in the case of the small hill, $L \approx 10l_s$.

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