

Numerical investigation of flow separation in the lee side of transverse dunes

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Abstract

We investigate flow separation in the wind over transverse sand dunes. CFD simulations of the wind over differently shaped dunes are performed. The length of the recirculation region after the brink of the dune is found to depend strongly on the shape of the dune. A phenomenological expression for the separation length is presented. Suitably non-dimensionalised, it depends linearly on the angle of the dune at the slip face brink. We propose a fit for the shape of the separating streamline, which is well approximated by an ellipse.

Keywords: dunes, fluid mechanics, aeolian sand transport, flow separation

1 Introduction

Dunes are naturally occurring, beautifully shaped sand deposits. Since the middle of the previous century, they have attracted the attention of scientists who have been seeking to model them and understand the processes leading to their formation. From the point of view of the physicist, sand dunes constitute a variable boundary problem: The air flow is determined by the shape of the dune and in turn influences the dune shape by transporting sand grains. Therefore the air flow over dunes is of great importance for understanding dune formation and evolution. Consequently, this topic has aroused a great deal of interest since the days of Bagnold [Bag41, Bag51] and led to a significant number of publications [Ben59, Sut67, BH79, Haf83, MS86, RH87, Woo95, NS89, WHC⁺91, AH04].

Since the start of scientific interest in dunes, there has been some work on the topic of flow separation in the lee of dunes, both theoretical [NS89, PWW04a] and experimental (e. g. [Eng81, SK90, RIR96]). However, due to the difficult nature of the problem, these papers have only tackled part of the problem. In several publications, transverse dunes have been modelled as triangular structures [Eng81, PWW04a, PWW⁺04b]. Field measurements of air flow over dunes, on the other hand, tend to lack measurements of the dune profile [SK90, FK96].

A recent field measurement [PSH⁺04] suggests that the shape of transverse dunes has significant influence on the length of the recirculation region. Since the sand transport in the recirculation region in the lee of a dune is negligible, the foot of the following dune shape is located at or downwind of the flow reattachment point (if one assumes the dune shapes to be stable). Therefore the distance of closely spaced dunes is a limiting measure of the length of the recirculation region. In reference [PSH⁺04] the separation length after different dunes was determined in this way.

In this work we will present results for widely spaced or isolated transverse dunes. This is to some extent an idealisation. However, we think it is a useful idealisation: We want to concentrate on the effect of the dune shape of a single dune, and taking

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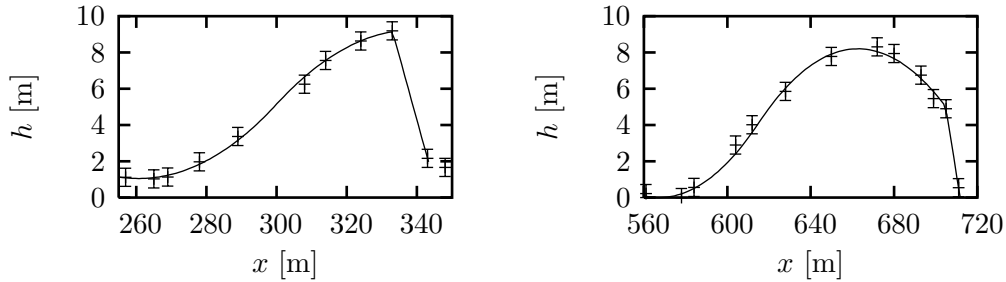


Figure 1: Realistic profiles of transverse dunes can be described approximately by two circle segments. Data from [PSH⁺04].

into account the presence and shape of neighbouring dunes would introduce additional parameters. In Section 5 we discuss the effect of considering transverse dunes which are part of a dune field.

This text is organised as follows: In the following Section 2 the models and parameters of our CFD simulations are described. The geometry of the dune shapes we modelled is also presented there. Section 3 presents our results for the length of flow separation and the phenomenological formula we found. In Section 4 the shape of the separating streamline extracted from the simulation is modelled mathematically. In Section 5 we briefly discuss the situation of a transverse dune in a field of closely spaced dunes. Section 6 compares our results with previous work. The last section presents a summary.

2 Method

Our simulations were performed with the computational fluid dynamics software FLUENT [FLU05]. This software simulates the Reynolds-averaged Navier-Stokes equations complemented by a turbulence model. The simulations were two-dimensional. This implies translationally invariant dune shapes, i. e. perfectly straight transverse dunes, and a wind direction perpendicular to the dunes. The simulation grid was square. We refined it near the ground to allow a modelling of near-wall flow as accurate as possible using wall functions. Second-order discretisation schemes were used for all quantities for which this was possible.

Besides the Reynolds-averaged Navier-Stokes equations, an additional set of equations called the turbulence closure is required to determine a solution. We use the k - ϵ model with renormalisation group (RNG) extensions. This variant of the k - ϵ model was found to yield the most accurate results in flow separation situations [LL94, BBL⁺98, WN02].

The cross sections of the dune shapes were constructed from two circle segments, a concave one modelling the foot of the dune and a convex one for the crest. This shape was chosen for reasons of convenience — the program we used to create the geometry supports circle segments. But as can be seen from Figure 1, our shape provides a reasonable fit for real dunes. The figure displays the dunes number 4 and 7 from Reference [PSH⁺04]. Given the great variety of shapes found in nature, the measured shape may not be universal. But our geometric construction reflects the fact that the dune profile is curved upward at its foot and downward at its crest and therefore constitutes an improvement over the triangular shapes used previously.

To obtain different shapes, the position of the slip face was varied from the start to the end of the convex part, see Figure 2. Note that this has the consequence that not

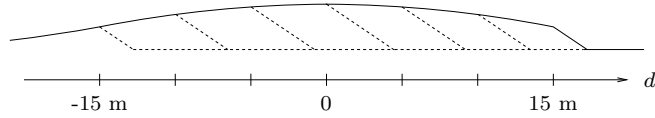


Figure 2: The seven different dune shapes investigated. The scale displays the brink position d . The crest height of the dunes with positive brink position is 3 metres; the height of those with negative brink position equals the brink height, which is the smaller the more negative d is.

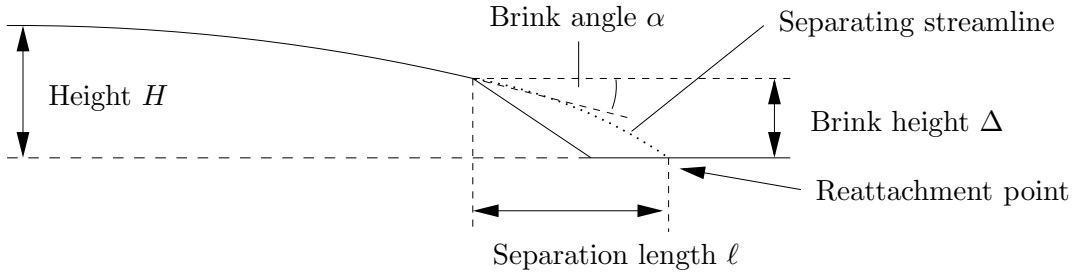


Figure 3: The geometric variables characterising the dune shapes. The brink angle is positive for dunes with a sharp brink and negative for round dunes as the one shown in this figure.

all the dunes have the same height. The heights and other geometrical data are given in Table 1.

The simulation results for the length of flow separation, our main quantity of interest, was found to depend slightly on the spacing of the simulation grid. To account for this small grid dependence, we performed the simulation of the flow over each dune with three different grid sizes and extrapolated the separation lengths to the continuum. The average grid spacings were 10, 7 and 5 cm, respectively.

The region around the dune in which the flow was simulated was chosen large enough so that the boundaries did not influence the results. This was verified by performing simulations with larger simulation areas for some dune shapes and comparing the results. The simulation region extends 45 m to the left and 70 m to the right from brink position 0 (see Figure 4). The height of the simulated region was chosen to be 30 m for all dunes except the one with the most negative brink position which had the smallest height, where 20 m was found to be sufficient.

The velocity profile at the influx boundary of the simulation region was set to the

Brink position d [m]	Height H [m]	Brink height Δ [m]	Brink angle α [°]
-15	1.5	1.5	11.4
-10	2.337	2.337	7.6
-5	2.835	2.835	3.78
0	3	3	0
5	3	2.835	-3.78
10	3	2.337	-7.6
15	3	1.5	-11.4

Table 1: Geometric variables of the simulated dunes. See Figure 3 for a definition of the geometric variables. The brink angle is defined to be positive if the upwind slope is positive at the brink.

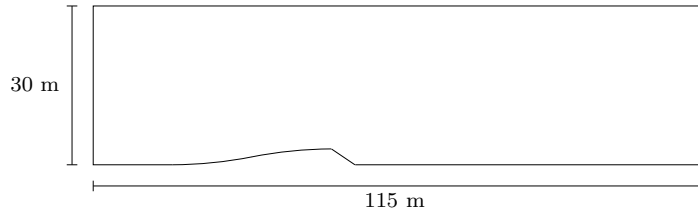


Figure 4: The simulation region around the dune.

Brink angle α [°]	Separation length ℓ [m]	ℓ/Δ
11.4	13.22 ± 0.5	8.81 ± 0.33
7.6	19.16 ± 0.5	8.20 ± 0.21
3.78	20.78 ± 0.5	7.33 ± 0.18
0	19.47 ± 0.5	6.49 ± 0.17
-3.78	15.90 ± 0.53	5.61 ± 0.19
-7.6	11.20 ± 0.73	4.79 ± 0.31
-11.4	5.91 ± 0.5	3.94 ± 0.33

Table 2: Results for the flow separation length. The error is composed of the statistical error of the determination of the flow reattachment point and a systematic error (see text).

logarithmic profile which forms in flow over a plane in neutral atmospheric conditions:

$$v(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_0}, \quad (1)$$

where $\kappa \approx 0.4$ is the von Kármán constant. The shear velocity was chosen to be $u_* = 0.4$ m/s. The size of the roughness elements on the ground, the grain size, was fixed to $250 \mu\text{m}$. The roughness length was chosen as $1/30$ of the grain size, $z_0 \approx 8.33 \mu\text{m}$ [Bag41, WSM97].

3 The flow separation length

The length of flow separation, our quantity of interest, was measured from the slip face brink, where the flow separates, to the flow reattachment point (see Figure 3), defined to be the position at which the velocity near the ground changes direction from against the flow to in flow direction. The separation lengths determined from simulations with different grid spacings were extrapolated to the continuum with the standard linear regression formulas. To non-dimensionalise the separation length ℓ , it was divided by the height of the slip face. Table 2 shows the results for all simulated dunes.

The error in the separation length was calculated as follows: The determination of the flow reattachment position for one particular simulation was accurate to one grid spacing. The corresponding error in the continuum limit results from the linear regression formulas. This error does not account for biases which may be inherent in the turbulence model, the kind of grid and the parameter settings used. We estimate that systematic error in the absolute separation length to be 0.5 m. The errors given in Table 2 are the result of adding these errors quadratically. The systematic error dominates in most cases.

Our main interest here is in the dependence of the separation length on the dune shape. We find that ℓ/Δ is larger for dunes with a sharp brink than for rounded dunes.

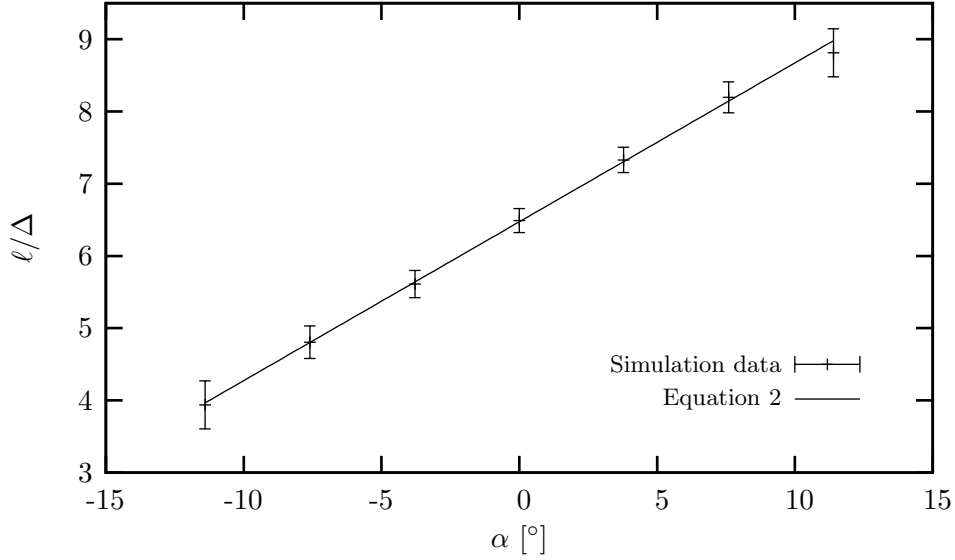


Figure 5: Dependence of the non-dimensionalised separation length on the angle α . The relationship is remarkably linear. Note that the rightmost value of α belongs to the dune with the sharpest brink, i. e. the shortest dune.

It depends linearly on the brink position d and on the angle of the dune shape at the brink, α . As can be seen in Figure 5, the linear relation holds for the whole range of brink angles investigated here. Fitting the relation

$$\ell(\alpha)/\Delta(\alpha) = A \cdot \alpha + B, \quad (2)$$

we obtain $A = 0.22/^\circ$ and $B = 6.473$.

To give the reader an idea of the actual separation lengths we obtained, we also give

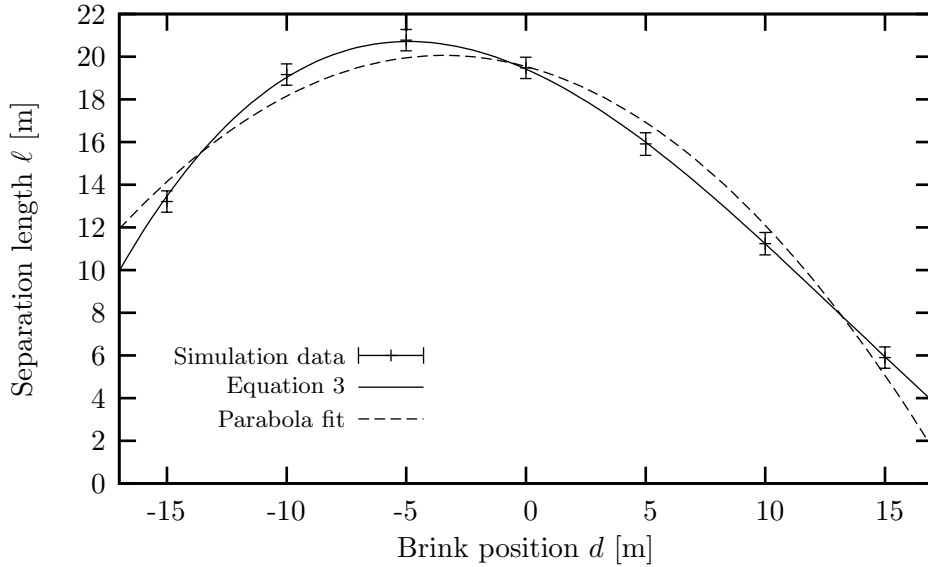


Figure 6: Dependence of the absolute flow separation length on the brink position. The expression derived from the linear angle dependence displayed in Figure 5 provides a much better fit than a parabola. Note that only the dunes with $d \geq 0$ have the same height, while the others become smaller with decreasing d (see Figure 2).

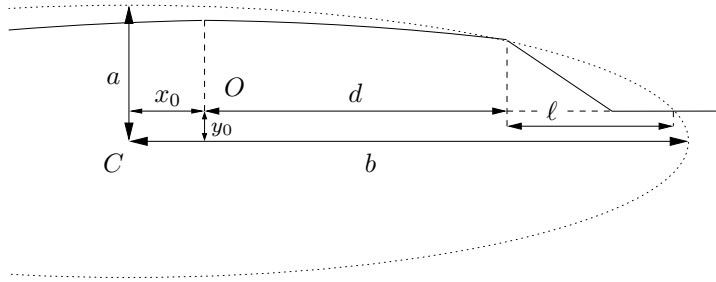


Figure 7: The parametrisation of the ellipse describing the separating streamline. In this example, $d > 0$ and both x_0 and $y_0 < 0$. C is the centre of the ellipse, and O is the origin, at ground level and at the horizontal position of the dune crest.

our results for the absolute separation length. The length of flow separation decreases both for large and for very negative brink positions. As one can see from Figure 6, the maximum does not coincide with $d = 0$ but lies to the left of that value.

We compute the absolute separation lengths from Equation (2) by using the geometrical relation between the brink angle α and the brink position d . This relation can be obtained from the geometry of our dune profiles described above.

$$\begin{aligned} \ell(\alpha(d)) &= (A \cdot \alpha(d) + B) \Delta(\alpha(d)) \\ &= \left(-A \arcsin \frac{d}{R} + B \right) \cdot \left(H_{\max} - d \tan \left(\frac{1}{2} \arcsin \frac{d}{R} \right) \right) \end{aligned} \quad (3)$$

This equation contains the crest height of the round dunes, H_{\max} , and the curvature radius of the dune shape at the crest, R . Both are quantities related to the set of dunes we study here, not single dunes, and therefore stay constant during our investigation.

For curiosity we can also try a different fit to the one in Equation 3 and compare their quality. As the data have a maximum and everywhere negative curvature, the most obvious candidate for a fit is a polynomial of second order, that is a parabola. It is plotted in Figure 6 but does not fit particularly well, even though it has three parameters compared to two for our fit. The angle-based fit (3) has a mean deviation of 0.28 compared to 2.2 for the parabola fit.

4 The separating streamline

In order to model the formation and evolution of sand dunes, it is necessary to calculate the ground shear stress on which the flux of transported sand crucially depends. While analytic derivations of the shear stress on landforms exist [HLR88, WHC⁺91], they apply to round hills from which air flow does not separate. The sand flux over dunes has been computed without taking into account flow separation by Weng et al. [WHC⁺91]. One can go one step further and compute the shear stress over a shape which for the most part follows the dune shape, but coincides with the separating streamline in the region of flow separation [Sau01, KSH03]. Since the shape of stationary dunes depends sensitively on the shear stress, it is of great importance to know the shape of the separating streamline.

From our CFD simulation, we extracted the streamline which just touches the brink of the dune and which therefore represents a very good approximation of the separating streamline. In each case, we used the simulation with the smallest grid

spacing, 5 cm. The simulation streamline does not separate directly at the brink, but a small distance down the slip face. But since this distance amounted to two grid spacings in all simulations, independent of which grid spacing was chosen, this is a numerical effect due to the difficult numerics at the flow separation point. Therefore we aim to model only the part of the separating streamline which curves downwards, not the dip near the separation point.

We found that the shape of the separating streamline is well described by an ellipse. An ellipse is determined by four parameters, the coordinates of the centre and the two semiaxes (see Figure 7):

$$\frac{(y - y_0)^2}{a^2} + \frac{(x - x_0)^2}{b^2} = 1 \quad (4)$$

Both the brink and the reattachment point have to lie on this ellipse, so there remain two free parameters which have to be fitted. We choose to fit x_0 and b and calculate y_0 and a from them using the position of the brink and the reattachment point. The brink is located at the point (d, Δ) , the reattachment point is $(d + \ell, 0)$. Putting these two points into the ellipse equation (4) and performing some algebra, we obtain a biquadratic equation for a :

$$a^4 + \frac{4 \Delta^2 b^2}{(2\delta + \ell)^2 \ell^2} \left[(\delta + \ell)^2 - b^2 - \frac{1}{2} (2\delta + \ell) \ell \right] a^2 + \frac{\Delta^4 b^4}{(2\delta + \ell)^2 \ell^2} = 0, \quad (5)$$

$$\text{where } \delta = d - x_0.$$

The biquadratic equation (5) can be solved with the standard formula. Choosing the solution for which the ellipse intersects the ground with negative slope, we obtain:

$$a^2 = -\frac{2 \Delta^2 b^2}{(2\delta + \ell)^2 \ell^2} \left[\dots \right] + \sqrt{\left(\frac{2 \Delta^2 b^2}{(2\delta + \ell)^2 \ell^2} \left[\dots \right] \right)^2 - \frac{\Delta^4 b^4}{(2\delta + \ell)^2 \ell^2}}, \quad (6)$$

where the expression in square brackets is the same as in Equation 5. Since a is positive by definition, it is thereby uniquely determined. y_0 can then be computed from a and the constraints, giving:

$$y_0 = \frac{a^2}{2\Delta} \left(\frac{\Delta^2}{a^2} - \frac{(2\delta + \ell) \ell}{b^2} \right). \quad (7)$$

It remains to determine the unknown variables in Equation 5. Besides the measures given by the geometry of the dune, the equation contains ℓ , b and x_0 . ℓ is given by Equation 2. The other two quantities have to be fitted. We obtain the best overall fit with the following expressions:

$$x_0 = \begin{cases} 0 & d \geq 0, \\ -7 (H_{\max} - \Delta) & d < 0 \end{cases} \quad (8)$$

$$b = (d + \ell - x_0) + 0.04 \cdot H_{\max} \quad (9)$$

It is clear that the difference between the x coordinates of the ellipse's centre and the reattachment point, $d + \ell - x_0$, is a lower bound for the horizontal semiaxis. The additional term in Eq. 9 is required for the ellipse to intersect the line $y = 0$ at an angle rather than vertically. It does not depend on the brink position.

The fit together with the data from both sets of simulations is displayed in Figure 8. It can be seen that the fit is very accurate. The upward curvature of the simulation streamlines close to the brink is associated with the delay of flow separation by two grid spacings. This is a numerical effect which we do not model.

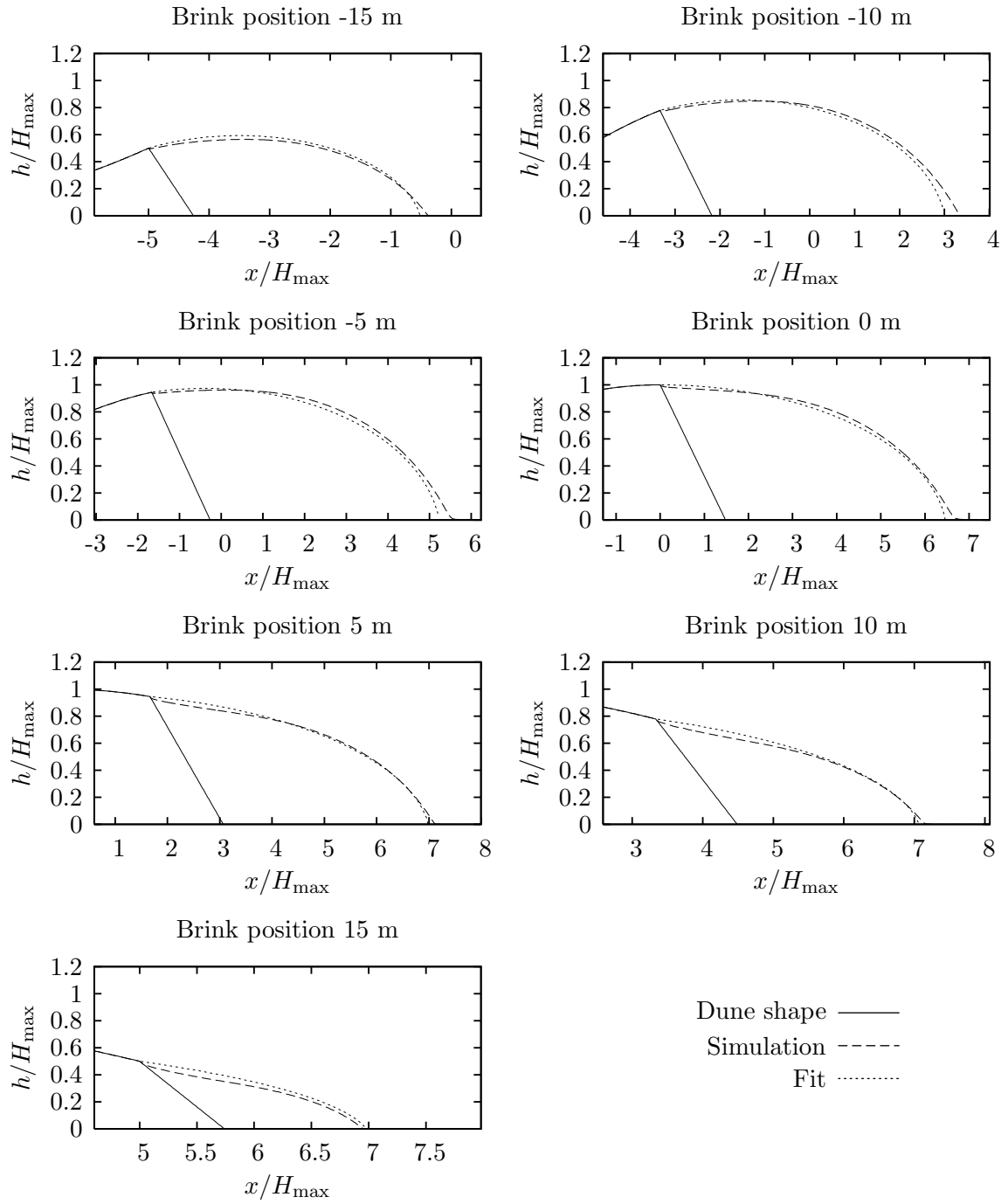


Figure 8: Fit for the separating streamlines. All coordinates are rescaled using the maximal height of the dunes, 3m.



Figure 9: The simulation region in the simulation of multiple dunes.

Dune number	1	2	3	4	5	6	7	8	9
Separation length	17.77	15.78	15.41	15.19	15.00	14.90	14.80	14.71	14.71

Table 3: The separation lengths obtained from the simulation of closely spaced dunes. The errors are a statistical error of 0.05 m in the determination of the reattachment point and a systematical error of 0.5 m.

5 Closely spaced dunes

In the previous sections we have considered single transverse dunes. This was done to be able to make a statement about the shape dependence of flow separation without at the same time dealing with complications due to potential neighbouring dunes. In reality, this corresponds to the case of isolated dunes, which have a distance to their neighbours of around three times their length or more.

To get an idea of the influence of neighbouring dunes close by, we performed a simulation of closely spaced dunes. The shape of the dunes was the same as for $d = 0$ in Figure 2. The dunes were set next to each other so that the foot of the upwind slope of each following dune coincided with the slip face foot of the previous one, as shown in Figure 9. The simulation parameters were the same as previously. This simulation was only done with the grid spacing 0.1 m.

It should be understood that our geometrical construction leads to the upwind side of a dune rising immediately at the foot of the previous dune. Since the sand cannot be moved within the separation region, this means that this profile is not stable. However, as we can know the length of flow separation only after our simulation, we cannot know in advance what a stable profile would look like.

Table 3 shows the separation lengths in the lee of the nine dunes in the simulation.

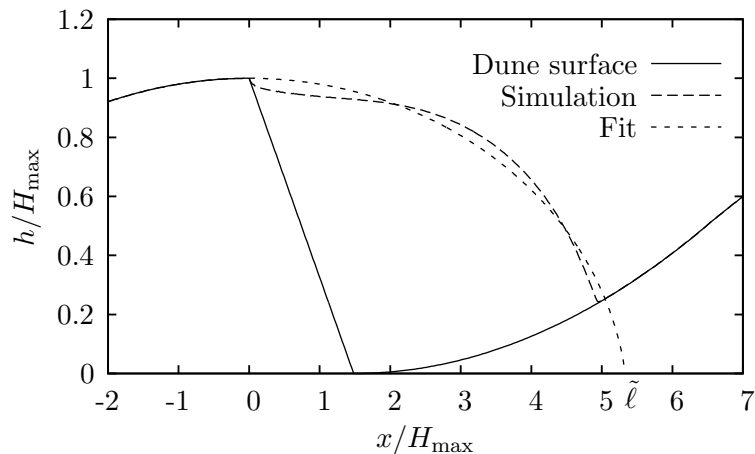


Figure 10: The ellipse fit also describes the separating streamline of closely spaced dunes. This Figure shows the last of the dunes in Figure 9. Both h and x are normalised by dividing by the crest height, 3 m.

One can see that the values converge towards the downwind end of the simulation area. Therefore we take the separation length of the last dune as the value for a dune in an extended dune field. By comparison with table 2, it is $\approx 25\%$ smaller than for an isolated dune.

We can now fit the separating streamline in the same way as for the isolated dunes. The Formulas (4) to (9) apply unchanged except for one modification: Since the foot of the downwind dune curves upwards, the separating streamline intersects the ground upwind of where it would for an isolated dune. We account for this by replacing the separation length ℓ by an $\tilde{\ell}$, which is larger than the separation length. $\tilde{\ell}$ is the intercept of the separation bubble shape with the line $h = 0$, while for closely spaced dunes the reattachment position at $x = \ell$ has a height $h > 0$. The best fit is obtained for $\tilde{\ell} = 16$ m. It is shown in Figure 10. The upward curvature of the simulated streamline close to the brink is especially pronounced because of the large grid spacing of this simulation.

6 Discussion

This section compares our results to previous work. A recent review of air flow over transverse dunes [WN02] cites values of 4–10 for ℓ/Δ . Our results also lie within that range (see Figure 5). Engel [Eng81] finds values for the non-dimensionalised separation length between 4 and a little over 6, depending on the roughness and the aspect ratio of triangular dunes. In Reference [PWW04a] a wide range between 3 and 15 is given for the same quantity. Their values are for an aspect ratio of 0.1, which applies to our dune with $\alpha = 0$, are 5.67 and 8.13, depending on the height. This compares well with our value of 6.49. The discrepancy can be explained by the different shape, in particular the fact that our dune shape for $\alpha = 0$ has a horizontal tangent at the brink, whereas the dunes in Ref. [PWW04a] are triangular.

The field measurements [PSH⁺04] were performed in a closely spaced dune field. The dune profile was measured along a straight line in wind direction, perpendicular to the dunes. The authors find that the distance between the brink of each dune and the foot of the following one is typically four times the height or below. Under the assumption that the dune field is stationary, this distance is an upper limit of the separation length. We found a separation length of 4.9 times the height for closely spaced dunes with a horizontal tangent at the brink. Considering that only two of the six dunes measured in Ref. [PSH⁺04] had positive slope at the brink and that the dunes with the shortest separation length were indeed very round, the agreement is not bad.

7 Summary and outlook

We have investigated the air flow over transverse dunes of different shapes using the commercial CFD software FLUENT. The variation in shape was achieved by moving the position of the slip face of the dune to different places.

We have determined the length of flow separation in the lee of these dunes. For each dune shape, six simulations were performed, with two absolute sizes of the dune and three different grid spacings to be able to remove the remaining influences of the grid spacing. The maximal separation length does not occur for dune shapes with a horizontal tangent at the brink, but for shapes with a somewhat sharper brink. The separation length non-dimensionalised through division by the slip face height was found to depend linearly on the position of the slip face as represented in Equation 2. This linear law can be rewritten with the help of geometric properties of the dune to

give the absolute separation length.

The shape of the separating streamline, that is the boundary of the recirculation region, is well approximated by an ellipse. This ellipse is constrained by the requirement that the brink and the flow reattachment point lie on it. The horizontal position of its centre is at the crest for rounded dunes. For dunes with a sharp brink, it lies to the left of the crest of the rounded dunes, and its position is proportional to the difference in height between the round dunes and the sharp dune in question (see Equation 8). The horizontal semiaxis of the ellipse has to be chosen so that its rightmost point lies to the right of the flow reattachment point by 0.04 times the height of the rounded dunes, independent of the brink position.

Lastly, we have extended our investigation by simulating the flow over a field of ten closely spaced transverse dunes. Here we restricted ourselves to one dune shape with horizontal tangent at the brink. The separation length reached an asymptotic value behind the ninth dune, which was 25 % less than the value for an isolated dune.

There still remain many open questions concerning the air flow over dunes. The most obvious restriction of our results is that they were obtained for transverse dunes only. The three-dimensional shape of other dunes, for instance barchans, calls for a three-dimensional description of their recirculation region. Furthermore, it should be investigated how accurate the concept of a separation bubble is: It has been assumed for the purpose of sand transport simulations [KSH03] that the wind shear stress on a dune shape is the same as the shear stress over a shape composed of a dune and the recirculation region in its lee. While good results for dune shapes support this assumptions, it should be verified from fluid dynamics.

Lastly, the influence of the dune size should be investigated. The flow over dunes is fully turbulent and therefore scale invariant. However, if the dune is scaled up while the ground roughness and the inflow velocity profile are kept invariant, the separation length can change. This was found for instance by Engel [Eng81]. It bears investigation how the phenomenological laws and constants found in this work depend on the dune size.

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