Effect of texture on fracture of fibrous materials

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Abstract

We present numerical results for the effect of orientation and placement distribution of fibres on fracture processes in two-dimensional random fibrous networks. As can be expected, highest elastic modulus and strength can be obtained with bundles of oriented fibres. However, such an enhancement is obtained only at a cost of reduced strain-to-failure (i.e. the strain corresponding to the maximum in the stress–strain curve). We provide quantitative estimates of these effects and discuss micromechanical reasons for the observed differences in strength as well as in the number of failed connections prior to the macroscopic fracture. Generally, the strength of the network was found to be a function of the mean length ($l_0$) of fibre segments—parts of fibres between intersections with other fibres—whereas strain-to-failure apparently appears not to be affected by ($l_0$), but to be determined by the form of the segment length probability distribution. Finally, the form of the probability distribution for axial stresses is exponential independent of texture. © 1998 Elsevier Science S.A. All rights reserved.

1. Introduction

Physical systems in which random fibrous network forms the load-carrying backbone include ordinary commercial paper and certain polymeric products. The spatial and orientational location of fibres is of great importance in determining the mechanical properties of physical systems consisting of a purely fibrous network on one hand and composites with short fibres inside a matrix on the other. In the case of paper, the manufacturing process produces a non-uniform orientation distribution for the fibres, whereby elastic and fracture properties are different in the ‘machine direction’ from those in the ‘cross direction’ [1].

Another interesting application group involving fibres are short-fibre composites. In these materials, strong fibres are embedded in a continuous matrix. The fibrous structure may or may not percolate, depending on the volume fraction and orientation distribution of fibres. In most ‘traditional’ fibre reinforced composites, the fibres do not form a connected network. In certain metal matrix composites, the 3D fibrous structure (‘preform’) is a connected one. In addition, in modern materials such as C/C-SiC, the volume fraction of fibres is considerably larger than in the older ones (roughly 2/3 for C/C-SiC) and connected structures exist [2].

Theories for elasticity of non-woven purely fibrous materials such as paper [3,4] have been based on shear-lag type models for stress transfer, where a single fibre is embedded into an effective medium [5]. These fibre network versions of the theory were originally developed with homogeneous networks in mind, but orientational distributions can easily be incorporated in them. For isotropic, homogeneous networks, the Finite Element Method (FEM) has been employed to study numerically elasticity [6–9] plasticity [10] and, to some extent, fracture [11,9] of isotropically random fibrous networks (RFNs). The simulations have, e.g. demonstrated that shear-lag type theories which ignore axial stress transfer are insufficient in accounting for elastic properties of

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random fibre networks, let alone fracture properties [8]. In the results cited, fracture must be dealt with outside the actual FEM solver. We shall present an alternative way to study fracture in fibrous systems later on.

The quasi-static time scale fracture of two-dimensional generic disordered systems has been studied extensively using models numerically more feasible than FEM. These models are based on regular lattices (for a review, see [12]). In these models, disorder is typically due to either random dilution in the lattice [13] or a distribution for the breaking limits of the bonds between lattice points [14]. Topics which have been addressed with this kind of model include the finite size scaling of strength [13,14] and the effect of the strength of disorder on the roughness of rupture interfaces [15]. These models can be viewed as coarse-grained models for disordered solids.

In modelling fracture of random fibre networks, regular lattice models are insufficient because they do not reflect the geometrical type of disorder we want the model to have. An effect associated with geometrical disorder, the build-up of RFNs with random placement of fibres generates a coverage distribution which is Poissonian independent of coverage [16]. Also, the segment length distribution is exponential no matter how high the density of fibres is. Systems with fibres have been simulated: for example, brittle [17,18] as well as viscoelastic [19] fracture of fibre reinforced composites have been studied using numerical models, in which fibres are placed onto a regular lattice. In view of the goal of having real geometrical disorder in the fibrous system, also in these cases the fibrous structure is fairly regular.

In this article we address the question of the effect of texture of fibrous structure in networks in which the fibres form a mechanically connected structure using a truly geometrically random model which avoids some of the drawbacks in FEM-based approaches discussed above. The results are applicable to materials consisting of almost solely a fiber network such as paper, but also (to some extent) to 3d 'preforms' and to composites in which the volume fraction of fibres is high.

In Section 2 we define the numerical model used of the subsequent results in Section 3. In Section 3 the results of fracture simulations are presented. We conclude this work with Section 4.

2. Simulation model

Our numerical model for simulating the effect of texture of random fibrous networks on their fracture properties consists of two parts: the generation of the network and the actual simulation of fracture. These parts will be explained in the following. First, however, we comment on the choice of parameters in our model. We have chosen to simulate systems with a relatively small number of fibres per unit area (two times the percolation density), with 'slender' fibres (narrow compared to their length). This is due to the fact that at a low density of narrow fibres, the average width-to-length ratio of segments, or parts of fibres between intersections (nodes) with other fibres, is high and the placement of fibres has most effect on mechanical properties. This issue will be discussed in more detail in Section 3.

2.1. Generation of spatial structure

We generate a random arrangement of fibres on a two-dimensional xy plane of size $L \times L$, forming rigid junctions at points in which two fibers cross. The fibres are all of equal length $l_f$ and width $w_f = l_f/100$. The parameters which can be adjusted in the generation of the network are the total number of fibres, $N_f$, the orientation distribution of fibres, $P_o(\theta)$, and the spatial distribution of the centres of mass of fibres, $P_c(x)$. The details of the distributions used will be discussed in detail below, but first an overview of the process is given.

The fibres are placed onto the xy plane one by one, with their location and orientation drawn from the distributions $P_c(x)$ and $P_o(\theta)$, respectively. After all the fibres have been placed onto the simulation area, the parts of fibres crossing the boundary of the rectangular area are moved to the opposite end of the system. This procedure also ensures that the average density of fibres at the border is the same as in the centre of the sample. Next, the locations of connections between fibres are computed and fibres are divided into segments between the points of connection with other fibres. The process of division of fibres into segments is followed by removal of redundant parts of fibres, i.e. ends of fibres which are connected to one node only and do not cross any of the four borders of the simulation area.

In all the following discussions, the number of fibres in the system corresponds to twice the percolation
with outside water on.

We have used three main schemes for the placement of fibres (see Fig. 1 for an illustration). The first is homogeneous both in orientation and location of the fibres, which we call the Random Fibre Network (RFN). In the next one, the location distribution is uniform but the orientation distribution is almost bimodal in such a way that fibres are oriented in directions close to x and y axes only. This scheme is called Random Biaxial Network (RBN). The last scheme, the Fibre Bundle Model (FBM), involves an almost biaxial orientation distribution and a placement of the fibres such that six bundles are formed both in x and y directions. This scheme has been inspired by the "biaxial" orientation of fibres in composites such as C/C-SIC. It must be mentioned here that in the case of the composite material mentioned the fibrous structure is of 3D cross-ply type, whereby the FBM scheme is a '2D approximation' of it.

The distributions for different schemes are summarized in Table 1. δ(x) signifies the Dirac delta function and d(x, w) the distribution

\[
d(x, w) = \begin{cases} 
\frac{1}{w}, & x \in \left[-\frac{w}{2}, 0\right] \\
0, & \text{otherwise} 
\end{cases}
\]

n is the number of fibre bundles for FBM. \(w_1\) and \(w_2\) are parameters controlling the width of angular distribution of fibres and the width of fibre bundles relative to their separation, respectively. The values \(w_1 = \pi/16\) and \(w_2 = 0.0625\) were used in the numerical studies presented later. A reduction of the width of the fibre bundles causes more fibres to be oriented in the direction of the main axis of the bundle (x or y axis) and also increases the number of short segments. Hence, one can expect strain-to-failure, or strain corresponding to the maximum of the stress–strain curve, to decrease with decreasing \(w\). Whether an increase in strength is achieved depends on how much exactly strain-to-failure is reduced.

Because the division of fibres into segments depends on texture, also the number of the segments depends on the placement scheme chosen although the total number of fibres in the system is the same for all the three schemes. The choice of the scheme also affects the fraction of fibre material removed in the optimization phase ('dangling ends'). Table 1 compares the aforementioned quantities for the three schemes. Obviously, the scheme 'FBM' makes best use of fibre material, and has also the largest number of fibre segments. In Fig. 2(a) the 'short segment' end of segment length distributions \(N(l)\) is shown for the three schemes. Clearly, FBM has much more segments than the other schemes, especially short ones. One should note that the segment length distribution for this scheme can be expected to be affected strongly by the width of the bundle and the width of the orientation distribution. Finally, we would like to recommend that the form of the distributions is a consequence of small system sizes used in the simulations. For larger values of \(L/l_p\), especially for high coverages, \(N(l)\) assumes a clearly negative exponential form (Fig. 2(b)). Note that for both \(L = 4 \times l_p\) and \(10 \times l_p\) decay of \(N(l)\) is slower for FBM than for the other two schemes and that FBM has an abundance of very short segments as compared RFN and RBN.

In previous work, hypotheses based on the form of probability distribution of stresses in segments, \(P(\sigma)\), have been put forward for plastic yielding [10] and fracture [6]. In this kind of argumentation, one assumes that plastic yieldings or fracture events are uncorrelated at low strains. It has been demonstrated numerically [6,8] that at high strains, the distribution of stresses in segments, \(P(\sigma)\), is exponential. One proceeds to assert that certain quantities in the initial phases of fracture, e.g. number of bonds which have yielded [10,21], can be predicted by integrating the high-end tail of \(P(\sigma)\). The exact connection between the geometry of the fibrous system (e.g. \(N(l)\)) and the stress probability distribution \(P(\sigma)\) is not known. For example, diluted lattice fuse models also produce exponential probability distribution for currents [22]. In Section 3 we shall demonstrate that texture apparently does not affect initial phases of fracture.

### 2.2. Simulation algorithm

The samples were (separately) subjected to both purely elongative and shear test, by applying adequate forces to the 'right' edges \(x = L\), cf. Fig. 1, of the system while holding the 'left' one \(x = 0\) fixed. The strain was increased by a constant amount \(\Delta \epsilon = 2.5 \times 10^{-4}\) for each fracture step.
Fig. 1. Three examples of arrangements of fibres in the beam model: isotropic network (RFN, a); biaxially oriented but randomly located fibres (RFN, b); fibre bundle model (FBM, c). System sizes displayed are $L = 10 \times l$.

The numerical model we use for obtaining the mechanical equilibrium is based on the beam model in regular lattice [23,24]. The beam model is an exact discretization of the linear Cosserat-type elasticity equations of a continuous material. Because our mechanical system is irregular, we use a generalized version of the beam
Table 1

<table>
<thead>
<tr>
<th>Scheme</th>
<th>( P_s(\theta) )</th>
<th>( P_s(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFN</td>
<td>const</td>
<td>const</td>
</tr>
<tr>
<td>RBN</td>
<td>((\delta(\theta) + \delta(\theta - \pi/2))/2)</td>
<td>const</td>
</tr>
<tr>
<td>FBM</td>
<td>((\delta(\theta, x) + \delta(\theta, x))/2)</td>
<td>( \frac{1}{n} \sum_{i=1}^{n} \delta(\theta, x_i) )</td>
</tr>
</tbody>
</table>

Fig. 2. Short length part of the segment length distribution for the three fibre placement schemes. \( q = 2q_s \); \( L = 4 \times L_s \) (a) and \( L = 10 \times L_s \) (b).

model [25]. In the general version, the lengths and the orientations of beams can be arbitrary. The difference to the aforementioned model is that since we do not wish to model elasticity of a continuous medium in a random grid but rather the mechanical behaviour of single elements, we do not scale the cross-sectional area of the fibre segments with their length. It should be noted that the beam model is based on an assumption of linearity of elastic interactions, i.e. the theory of small strains. Because a random fibre network includes very short segments, the strain of the entire sample at which the model is accurate is rather small. However, with a suitable choice of the fracturing limits one can ensure that strains will not transcend the domain of application of the small strain elasticity theory.

The network is relaxed to the mechanical equilibrium state using a Sequential Over-Relaxation (SOR)-type algorithm, in which the forces \( F_e \) acting on nodes are computed from the elastic forces \( F_{el} \) in the beams. The nodes are moved in the direction of \( F_e \)'s by an amount \( \varepsilon \delta x \), where \( \delta x \) is the displacement which would bring the beams attached to the node to a local mechanical equilibrium. The latter is estimated by multiplying the displacement due to elastic forces by the length of the shortest segment attached to the nodes. The stresses are relaxed and the iteration of mechanical equilibrium is continued until a predefined accuracy is obtained.

The fracture criterion for the fibre segments is of von Mises type [12, p. 233], i.e.
Above, $\varepsilon$ is the strain in a beam, $\theta$, its bending angle, $\xi$, the fracture limit of a fibre segment in elongation and $\theta_r$ that of a beam under bending. The values $\xi = 4 \times 10^{-3}$ and $\theta_r = 0.09$ radians ($5.2^\circ$) were used in the simulations.

After the appropriate fibre segments are broken in each fracture step, the stresses are again relaxed in the system, for all fibres the fracture criteria are checked, etc. This procedure is continued until the force needed to deform the system drops to zero. We wish to stress here that the scheme involving complete relaxation of stresses would correspond to quasi-static time scale fracture. A physical interpretation for this situation is a system in which the speed of sound in the material is much larger than the speed of crack propagation. Such a situation is expected to arise, e.g. in slow crack growth or in high cycle fatigue.

3. Numerical results

Our numerical data show clearly that the Fibre Bundle Model yields much higher strength than the other two schemes, but at the same time strain-to-failure is reduced with respect to them (Fig. 3). This observation applies both to elongative and shear loading modes. The large difference in strength between FBM and the two other schemes can be attributed to low density used in the simulations. Earlier simulations have shown that $q = 2q_c$ is not yet in the asymptotic linear density dependence regime of elastic modulus, but rather the $E(q)$ relation is of power-law form, with exponent larger than unity [6,8]. Shortly put, at $q = 2q_c$ the RFN and RBN schemes make bad use of available fibre material.

The data given in Table 2 provide one possible means of quantification of the effectiveness in which the schemes make use of the fibre material. One might, for example, argue that the number of bonds, being a measure of the mean segment length, is a measure of the ratio of elastic energies in elongative and bending modes. To this end, we have simulated RFNs which have the same number of nodes as the $L = 4 \times L_f$ FBM model with $q = 2q_c$, or $N_c = 980$. This number of nodes is obtained with density $q = 2.3 \times q_c$ in the RFN scheme. The results (Fig. 4) display a remarkable increase in strength, which still remains below that of the FBM model. The explanation lies probably in the fact that for RFNs the segment length distribution is exponential, but for FBM there is an abundance of very short segments.

The strength of the samples has a distribution which is asymmetrical, having a long tail towards large values of strength. The shape resembles that of a Weibull distribution, although we do not have enough data to comment on the precise form. From previous numerical studies it is known that deviations from Weibull behaviour seem to take place at a length scale of fibres in RFNs even at high coverages [7]. However, experimental tests involving sample sizes of linear sizes of 10 cm or more seem to indicate that the distribution of strength obey a Weibull form in the case of paper [26].

Owing to the fact that there are many very short segments at the crossings of fibre bundles in the FBM model, this model exhibits a rapid increase in the number of broken bonds as a function of strain (Fig. 5). In the two other schemes, damage is first diffuse, and then it is typically centred around the final fracture area (cf. [11]). This development is clearly seen in Fig. 5, where the curves for RFN and RBN show an acceleration in the number of broken bonds as a function of strain. The speed of growth becomes constant after breakages take place in the final fracture zone.

The fracture results show that the distribution of segment lengths is an important quantity when it comes to strength of a fibrous system. This is partly due to fact that in the FBM scheme with many short fibre segments, the mean aspect ratio (ratio of length and width) of segments is lower, whereby the relative importance of elongation with respect to bending is higher. Thereby, it can be argued that elastic energy lies mostly in elongative modes, and fracture takes place earlier because elastic energy is not evenly distributed among bending and elongation but is concentrated on elongation. From the comparison of RFNs with two different coverages (Fig. 4), one notes that for a RFN the strength $F_c$ is increased but strain-to-failure $\varepsilon_f$ is not decreased, at least to a perceptible degree. Hence, the above explanation, as such, is not sufficient to account for the differences between different schemes. Apparently, it must be deduced that it is the shape of $P(\xi)$ and not its


\[ F(\varepsilon) \]

\[ 2 \times 10^6 \]

\[ 1.6 \times 10^6 \]

\[ 1.2 \times 10^6 \]

\[ 1.2 \times 10^6 \]

\[ 9 \times 10^5 \]

\[ 6 \times 10^5 \]

\[ 1.2 \times 10^6 \]

\[ 8 \times 10^4 \]

\[ 4 \times 10^4 \]

\[ 2 \times 10^4 \]

\[ 0 \]

\[ 0.005 \]

\[ 0.01 \]

\[ 0.015 \]

\[ 0.02 \]

\[ 0.024 \]

\[ 0.03 \]

\[ \varepsilon \]

\[ \varepsilon \]

Fig. 3. Averaged force-strain curves for RFN (+), RBN (×) and FBM (*) in elongation (a) and shear loading (b). The insets show RFN and RBN separately. The curves are averages over 10 geometries.

Table 2

<table>
<thead>
<tr>
<th>Scheme</th>
<th># (Segments) ( N_s )</th>
<th># (Nodes) ( N_n )</th>
<th>( f_{\text{cm}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFN</td>
<td>1149</td>
<td>715</td>
<td>25%</td>
</tr>
<tr>
<td>RBN</td>
<td>912</td>
<td>576</td>
<td>28%</td>
</tr>
<tr>
<td>FBM</td>
<td>1581</td>
<td>979</td>
<td>19%</td>
</tr>
</tbody>
</table>

The number of fibre segments \( N_s \), nodes \( N_n \) and the fraction \( f_{\text{cm}} \) of fibre material in dangling ends. System size is \( 4 \times 4 \times L \), \( q = 2q_s \), i.e. \( N_s = 182 \). The numbers quoted are averages over five systems.
Fig. 4. Comparison of averaged force-strain curves for RFN and FBM at a density $q = 2q$, with the curve for RFN having the same number of nodes as FBM (RFN2).

Fig. 5. Number of broken bonds as a function of strain for the three fibre placement schemes for elongative loading.

Fig. 6. Stress probability distributions for RFN (solid line) and RBN (dashed line) as well as FBM (inset). The curves are averages over three geometries for purely elastic loading.
average that counts in the case of $\epsilon_p$, whereas for $F_f$ the mean of $P(\epsilon_f)$ is a relevant quantity. However, the functional dependence between $F_f$ and $P(\epsilon_f)$ is not known.

We have studied the probability distributions of axial stresses, $P(\sigma_f)$, in all segments in the elastic regime to find out in which way, if any, texture has an effect on it. For our small networks, $P(\sigma_f) \sim \exp(-\sigma_f)$ in all cases for large $\sigma_f$ (Fig. 6). Hence, the texture of the network does not appear to affect the shape of the stress distributions, or this seems to be the case, at least, for the high stress end of the distribution. As can be seen by comparing Fig. 6(a) and (b), the scale of individual stresses is, however, affected very much by texture. The fluctuations at the high-stress end of some of the distributions are relatively large only due to small system size.

4. Discussion

We have studied with computer simulations the effect of the placement of fibres on fracture. More precisely, we have studied networks in which the density is low and the aspect ratio of the fibres is high, i.e. fibres are 'slender'. Three different placement schemes were compared: isotropic placement on one hand with both homogeneous and biaxial angular distributions and fibre bundle model on the other. Numerical results indicate that in these cases, the bundle model yields a strength clearly superior to the two others. At the same time, strain-to-failure is clearly lower in the model with fibre bundles as compared to those without. By comparing networks with equal number of fibre segments, the strength of the network was found to be a function of the mean segment length, whereas strain-to-failure appears to depend on the form of the segment length probability distribution.

Texture does not appear to affect the form of the high end of stress probability distributions, which is exponential in all the three cases studied. The exponential form of distributions has been previously employed to develop arguments for the behaviour, e.g. number of segments which have broken or yielded plastically in the initial (non-correlated) phases of plasticity or fracture. On the basis of our results, it appears that the applicability of this argumentation is not affected by texture.

Due both to the fibrous nature of the system and (for RFN) the Poissonian distribution of coverage [16], it would be interesting to study finite size scaling of strength in fibrous networks. It is possible that the distribution of local density from a location to another in the system would give rise to a special form of scaling. However, precisely due to Poissonian nature of geometry variations, the correlation length $\xi$ (used here in the sense of a measure for the size of a representative sample) is typically larger than the fibre length. Hence, the distribution of strengths can be expected to be quite wide for small system sizes. It would be difficult to get reliable statistics for the finite size scaling of strength.

Acknowledgments

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References