

Granular materials are found in industries as diverse as construction and pharmaceuticals, but we still do not understand their unusual properties

## Grains of understanding

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IF YOU watch with a physicist's eye a child playing in a sand pit, you might discover some surprising behaviour. The surface of the sand can be easily deformed, while the loose sand can be poured like a liquid. Yet dense sand is so strong that it is possible to drive a car on a beach. Sand also often moves in "bursts", such as when avalanches move down the slope of a sand pile. Once an avalanche reaches the bottom of the pile, it immediately loses its energy and comes to a complete standstill.

Sand flows in a different way to common fluids. Indeed, an hour glass only works because the speed of the flowing sand is constant with time. If water was used, for example, the velocity would fall as the water level in the upper part of the glass decreased. Granular media show many other strange effects. In 1831 Michael Faraday discovered that a vibrating layer of sand separates into several small heaps. The sand rises up in the centre of each peak and then flows down the surface. What stabilizes the heaps against the vibration, and what forces push the sand up in the centre? We still do not know.

Granular media crop up in many practical situations. Foodstuffs like flour and sugar are granular in nature, as are building materials such as rocks and cement. The powders that make up pharmaceutical pills, as well as the finished tablets, are also granular media. Such materials have been studied for most of this century to help improve industrial processes, but fundamental problems remain. We still do not understand how pipes carrying these materials become clogged up or why grains of different sizes separate into distinct bands.

Physicists first began taking an interest in the exotic properties of powders in the 19th century, and the likes of Faraday, Reynolds and Hagen made important contributions to our understanding of these fascinating materials. However, they were unable to come up with suitable theories for the deformation or flow velocities of granular materials, and soon turned their attentions elsewhere. It is only recently that physicists have shown renewed interest in granular materials. New concepts to describe collective phenomena in disordered media have been developed, including the idea of "self-organized criticality" introduced by Per Bak of the Brookhaven National Lab in the US in 1987 (see *Physics World* 1993 December pp39-43). Physicists are now applying such ideas to the practical problems of granular media.

Modern computers are also allowing us to simulate many-particle systems and to compare the results with experimental observations. For example, the technique of molecular dynamics means that we can calculate the movements of about ten thou-

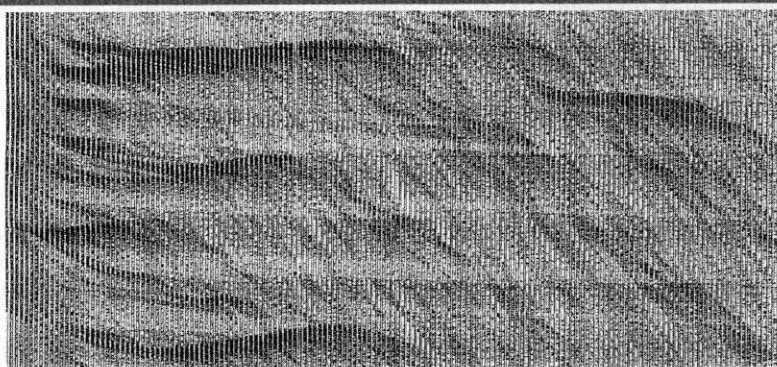
sand individual grains over periods of several minutes. Even more particles can be simulated by using simplified models defined on a regular lattice.

Many theories have been proposed in the last few years to describe the unusual properties of granular media. We can expect that many more will follow, particularly since some novel ideas will be needed to understand these mysterious materials.

### Grains lose energy

What is so special about granular media? A key feature is that they lose energy: collisions between the grains, perhaps due to vibrations or deformation of the surface, are inelastic and so dissipate energy. In contrast to microscopic systems, the energy is lost locally and the excess heat is radiated outwards or blown away by the surrounding air. This means that a system will stop moving if it is not constantly supplied with energy. Other dissipative systems include car traffic and the collective motion of living

### 1 Particles in a pipe



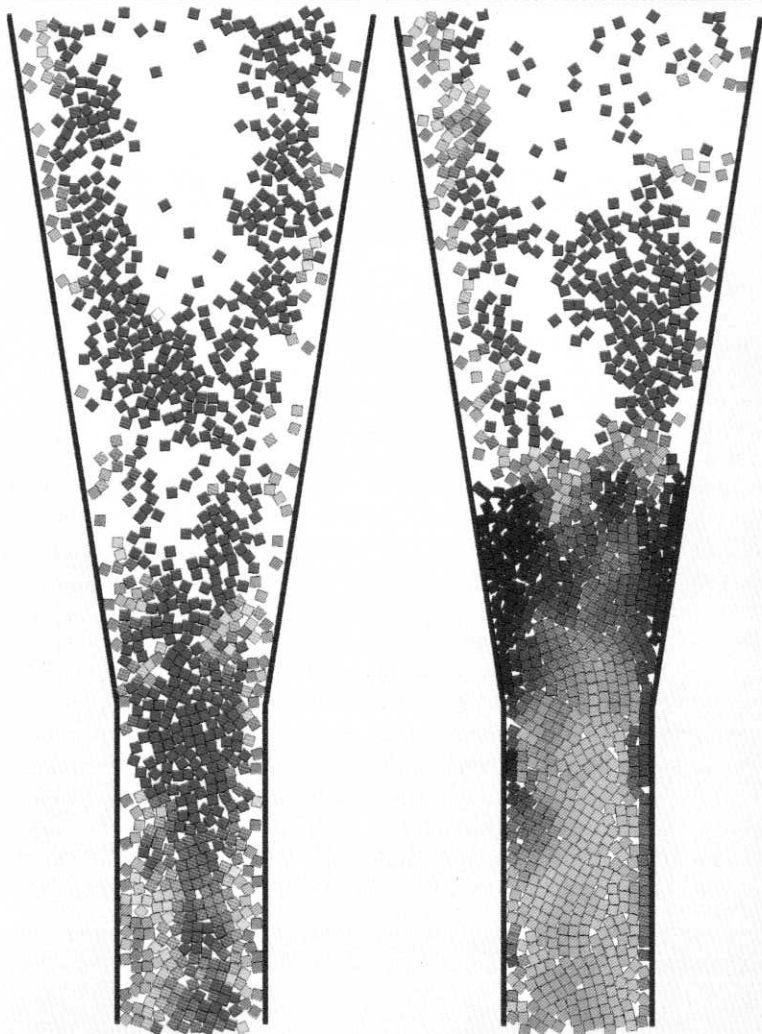
A computer simulation shows how small particles (the black dots) fall down a vertical pipe under the influence of gravity. The same vertical pipe is shown at intervals of 0.1 s at 27 different times, with time increasing towards the right. The vertical axis shows the position along the pipe, which is five times wider than the diameter of the particles and 350 times as long. The results show that the particles form dense clusters separated by empty regions. The simulations were performed by Thorsten Pöschel of the Humboldt University in Berlin.

species such as bacteria, fish and birds.

This energy loss means that the grains in flowing sand have velocities that are comparable with the overall flow velocity of the sand. As a result, we cannot separate the random, or "thermal", motion of the grains from the hydrodynamic motion of the material, and the viscosity is not constant – it increases with the shear velocity. This law, known as Bagnold's law after the English brigadier who discovered it in 1954, means that sand is a non-Newtonian fluid.

The loss of energy has another important consequence. If one

## 2 How silos get clogged up



Molecular dynamics simulations of equal-sized particles in a silo. The colours indicate the velocity of the particle, with red indicating fast particles and blue indicating slow ones. The grains flow easily when their flux is kept constant (left), but permanent clogging occurs when the flux is varied (right), even though the average flux of grains is the same.

region of the material has a slightly higher grain density than its surroundings, it will experience more inelastic collisions. This reduces the kinetic energy in that area, causing the local pressure to fall. A pressure gradient forms, so more grains flow into the region and it becomes even denser than before. Such a process can lead to the spontaneous formation of large density anomalies in an otherwise uniform material. When granular materials flow through pipes, for example, such density fluctuations travel as kinematic waves that are reminiscent of traffic jams. These waves can cause huge problems when emptying industrial silos: the forces they exert can be so large that “silo-quakes” can cause entire industrial installations to crumble.

K Schick and A Verween from the University of Leiden in the Netherlands have shown that these fluctuations have similar properties to earthquakes. The system drives itself to produce critical fluctuations in the density – this “criticality” means that waves of arbitrarily large wavelength can appear. A computer simulation shows how small discs cluster together when falling down a vertical pipe under gravity (figure 1). It shows that large dense clusters form between regions that are almost empty and nearly at rest. The lines of constant slope correspond to kinematic waves. Similar patterns are seen in traffic jams: regions of

dense traffic are separated by areas where the traffic flows more easily.

The density fluctuations can be analysed in more detail by calculating their power spectrum, which measures their frequency content at a given position in the pipe. The power spectrum decays with frequency as a power law, which indicates that fluctuations can appear on any timescale. Models show that the exponent of the power law is almost universally  $4/3$ , but we do not understand why this should be. Closed systems, such as vertical pipes, also show a peak at a characteristic frequency that corresponds to the wavelength of the kinematic waves.

In certain circumstances – such as when the pipe is too narrow, the grains are too dense, or there is too much friction against the walls – a pipe or silo can become clogged. This is catastrophic for industrial processes, but very little can be done to predict precisely when clogging will happen. Numerical simulations can help. For example, simulations of a two-dimensional silo show the effect of changing the rate at which grains are poured into the silo. The grains flow smoothly when they are poured at a constant rate (figure 2). However, the silo becomes clogged when the flow rate is more sporadic, even though the average flux remains the same.

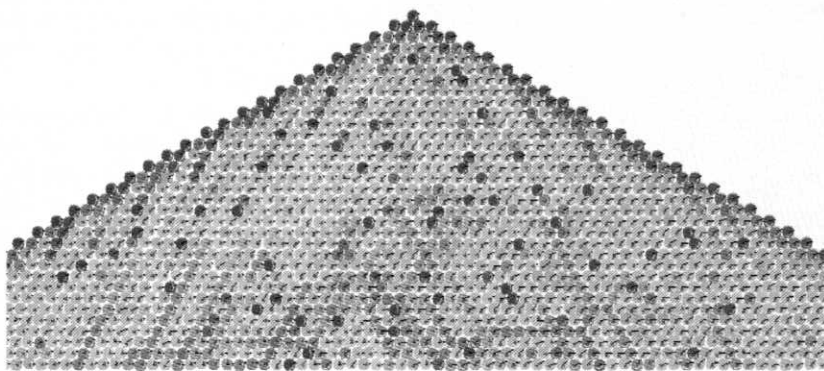
### Forces in piles of sand

The forces inside a sand pile act in quite a special way. In the ideal case of dry sand, the grains do not attract each other, so they do not resist the forces that pull them apart. Under compression, however, the forces between the grains depend on their shape. Spherical grains, for example, repel each other with a force proportional to  $\delta^{3/2}$ , where  $\delta$  is the displacement. This relationship, known as Hertz’s law, is based on the fact that the contact area of elastic spheres increases when they are pushed together. It is nonlinear for even small displacements, and other nonlinear expressions can be obtained for more complex grain shapes. In addition, friction acts between the grains when they move against each other.

When a sand pile is stationary, the weight of the grains in the upper part of the pile is transferred downwards along “force-lines”. These are essentially lines of touching grains that transmit the forces to the bottom of the pile. The orientations of the force-lines depend on how the pile was built. In 1981 J Smid and I Novosad of the Institute of Chemical Processes in Prague produced sand piles by continuously pouring sand grains on the apex of the pile. They found that the pressure at the base of the pile showed a pronounced minimum below the apex, and the precise origin of this “dip under the heap” continues to puzzle experts.

Sam Edwards and collaborators at Cambridge University in the UK have proposed an intuitive model for the process. They suggest that arches form inside the pile, which transfer the forces to the surface. Following this idea, Joachim Wittmer and colleagues at Edinburgh University developed the fixed principle axis (FPA) model, in which the principal axis of the stress tensor has a fixed orientation relative to gravity. The Edinburgh researchers claim that such a fixed orientation results from the fact that the pile was built by pouring grains down its surface.

### 3 Underneath the arches



This molecular dynamics simulation shows the forces inside a regular two-dimensional heap. The pile consists of 1717 discs with a random variation in size of about 5%. As the pressure increases, the colour of the particles changes from blue to green to red.

This seems a bold assumption, particularly since the elastic deformation of the particles is not fully taken into account, but the FPA model was the first to reproduce the dip under the heap quantitatively.

Numerical simulations have also helped us to understand the dip under the heap. A simulation by Stefan Luding here at Stuttgart shows the force-lines inside a regular sand pile (figure 3). The arching effect proposed by Edwards, which transfers pressure from the inside of the pile to the surface, can clearly be seen. A complex force network is also visible.

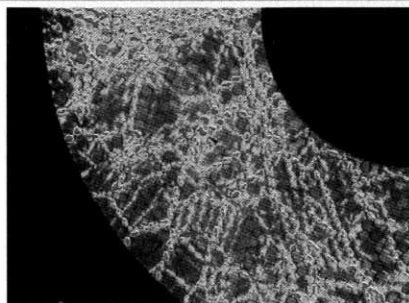
The force network can also be dramatically visualized in experiments that use a material like plexiglass, which can rotate the optical axis of light when under stress. If small discs of such a photoelastic material are put between two crossed polarizers, regions under stress will light up. Recent images obtained by Bob Behringer and colleagues at Duke University in North Carolina clearly reveal force networks in a shear band (figure 4).

Of particular interest are the variations in the forces that a granular material exerts on the walls of a container. The network of force-lines can concentrate the stresses at specific locations, while the packing structure of the grains can completely screen other regions of the wall from the forces. In the last few years, several researchers have proposed stochastic models to describe the action of these forces. For example, the “q-model” suggested by Sue Coppersmith and collaborators from the University of Chicago simplifies the forces to scalar quantities that are transferred downwards from layer to layer in a stochastic way. The simplicity of the model has the advantage that it can be solved exactly: it yields a distribution of forces that follows a power law at small forces and an exponential law at large forces. Although the model is relatively simple, its predictions at large forces show good agreement with experiments. However, at small forces different theoretical models predict different distributions, and experiments have yet to settle the dispute.

### Particles exert themselves

Compared with fluids, the particle nature of granular media

### 4 Stresses light up



These stress patterns were produced by putting plexiglass discs 9 mm across between two concentric wheels rotating against each other. Unstressed regions are blue, while stressed regions are coloured.

In a rotating drum the grains segregate into stripes perpendicular to the drum's rotation axis, while grains in free fall tend to segregate along the direction of gravity. Although segregation is usually a nuisance that we seek to avoid, we still do not understand how to prevent it.

### Dense sand under pressure

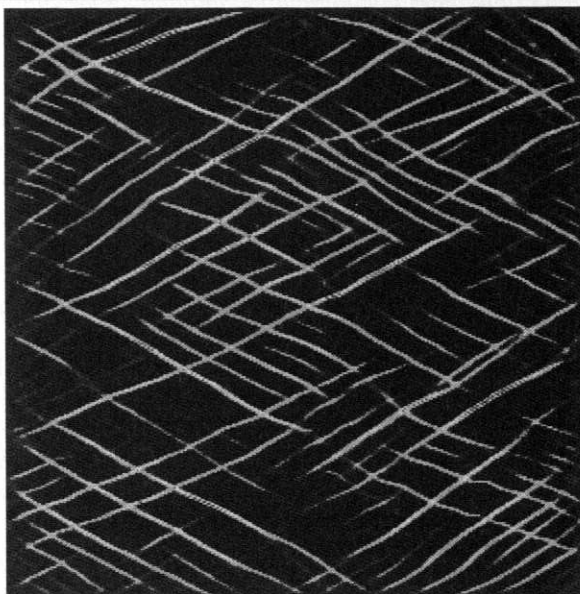
As we have already seen, the behaviour of a granular material is crucially determined by its density. In the case of both high and low packing densities, reasonably useful continuum theories that make it possible to predict the behaviour of the system have been developed in the last 30 years or so. For example, to calculate the deformation of a densely packed granular material like soil, it is possible to use a particular type of plasticity theory. This essentially provides a set of nonlinear differential equations that can be used to determine the stresses at which the soil will yield. In contrast to the plastic deformation of metals, the deformation of the soil is characterized by a volume change called the dilatancy.

An experiment first described by Reynolds in 1885 helps to explain the concept of dilatancy. Imagine a balloon filled with sand and water, with a glass pipe to measure the water level. When the balloon is deformed the water level goes down – even though everyone knows that it would go up if no sand were present. This is because the density of the sand must fall below a critical value, called the Reynolds density, before it can be deformed. The sand is relatively mobile below the Reynolds density, but at higher densities the grains prevent each other from moving.

introduces extra degrees of freedom into the system – for example, the grains can have different sizes and shapes. A typical phenomenon is segregation, where different types of grains separate into distinct regions. Segregation happens in many practical situations, such as when the grains flow down an inclined chute or move inside a rotating drum. It can even be seen in rings of asteroids.

Segregation can be a major problem in industrial applications, since it is often important to produce homogeneous mixtures of cement, drugs and other granular materials (see *Physics World* April p26). A well known phenomenon is the “brazil nut” effect. If you mix two types of nuts in a can and hit it several times, the bigger nuts will always rise to the top. However, there is still no consensus about what causes this effect. Indeed, it seems that several competing mechanisms are acting at the same time. In 1986 Anthony Rosato and colleagues at the Carnegie-Mellon University in Pittsburgh proposed a kinematic process in which smaller particles in a vibrated material move more efficiently into the holes that are created under large particles. This acts to push the larger particles upwards. Meanwhile, the Chicago group showed that in tall containers the driving force is convection. Another model proposed by Emanuele Caglioti and co-workers at the Ecole Supérieure de Physique et de Chimie Industrielles (ESPCI) in Paris elucidates the role of the different types of particles. It could also be that the larger particles become more mobile due to dissipation effects – they effectively become hotter and rise to the top.

## 5 Patterns in granular materials



This numerical simulation shows a typical network of shear bands in a granular solid. This model is based on a square grid of size  $300 \times 300$ . The colour scale indicates the local shear velocity – red indicates strong plastic deformation, while blue indicates that the deformation is elastic. (A Poliakov et al. 1994 *Fractals* 2 567)

In the balloon, the sand particles can only move past each other if the sand expands enough – the sand rises above the surface of the water, so the water level falls. The same effect is seen on the beach: if you step on a wet part of the beach, the sand around your foot becomes dry because the sand moves up around the foot, rather than being pushed down under its weight.

The application of these theories goes far beyond soil mechanics. The theories can also describe the slow motion of dry grains in industrial silos, for example, and can be used to calculate the surface shape of granular systems under external forces. One of the effects that plasticity theory can explain is the formation of “shear bands”. If the walls of a box full of sand are slowly moved parallel to each other, parallel lines form on the surface of the sand that correspond to shear planes inside the bulk. The shear planes have a lower density than the bulk, which means that they are more easily deformed. A numerical solution of the theory by Alex Poliakov and collaborators at the Jülich Research Centre in Germany shows a complex network of shear bands (figure 5). The simulation assumes that the vertical boundaries are being pushed together while the top and bottom boundaries are being pulled apart with the same velocity. It is based on a square grid: as the grid is made finer, the shear bands become thinner and more numerous. It seems that the system drives itself into a critical state, and the density of shear zones increases with the grid spacing as  $l^{0.58}$ , where  $l$  is the grid spacing. In other words, the network is fractal, and the system cannot be characterized by a specific length- or time-scale.

However, plasticity theory is only valid when the granular medium is densely packed. Deviations are common when calculating phenomena such as the clogging of pipes, which generally arise in less densely packed materials.

### Other theoretical approaches

The case of low densities can be treated by another theoretical approach. If the sand is stirred up to such an extent that the grains are separated by a finite distance, it is possible to formulate a model similar to the kinetic theory of gases. This has been done by Peter Haff of the California Institute of Technology, Jim Jenkins at Cornell University and Stuart Savage at McGill University in Montreal. This theory reproduces Bagnold’s viscosity and yields realistic velocity profiles for vibrated beds of powders, but it cannot correctly describe segregation or the characteristic “angle of repose” of a sand pile. This angle is the slope at which the grains in the pile would start to flow.

However, most of the intriguing phenomena in granular materials occur at intermediate densities, for which we have no reliable continuum theory. When sand is vibrated, poured or sheared, it will locally expand. However, dissipation means that it immediately compactifies again. This interplay generates a stop-

and-go behaviour, as illustrated by the avalanches seen on the surface of a sand pile. It also leads to the density fluctuations in a pipe that were described earlier.

These systems are characterized by threshold behaviour – in the case of sand piles, for example, avalanches of arbitrary size will happen once the angle of repose has reached some critical value. Moreover, the size of the avalanches, as measured by the number of grains in the avalanche, follows a power-law distribution in which large avalanches are less likely. Such systems cannot be characterized by a specific length- or time-scale. Their generic behaviour is now called self-organized criticality, since the system drives itself to some critical point, and it has been studied in simplified models by Per Bak at Brookhaven and collaborators. In granular materials, these ideas suggest that fluid-like regions of arbitrary size will appear once

the density goes below the Reynolds density. Indeed, experiments show that the size of avalanches on small piles of sand, and on piles of long rice grains, follow a power-law distribution.

Such power-law distributions have also been found in the sound frequencies emitted by sand that is sheared and in the frequency spectrum of density fluctuations in sand poured through a pipe. In contrast, avalanches on larger piles are usually characterized by a single size and frequency, which depends on the difference between static and dynamic friction.

### Future directions

The understanding of granular media still poses many problems. While theories in the limits of low and high densities give reasonable agreement with experiments, these theories cannot explain many of the phenomena that arise at the intermediate densities typical of real applications. One cause of these difficulties is the criticality of many granular systems. Another problem with using continuum theories is that granular media consist of many individual particles, which can lead to effects like segregation.

New ideas from statistical mechanics and nonlinear dynamics are now revolutionizing our understanding of granular materials. Given that these materials have immense practical importance in industry, we can expect that this revolution will continue for many years to come.

### Further reading

R P Behringer and J T Jenkins (ed) 1997 *Powders & Grains* 97 (Balkema, Rotterdam)

A Hansen and D Bideau (ed) 1992 *Disorder and Granular Media* (North-Holland, Amsterdam)

H M Jaeger and S R Nagel 1992 *Science* **255** 1523

H M Jaeger, S R Nagel and R P Behringer 1996 *Physics Today* April pp32–38

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