EXACT DETERMINATION OF FORCE NETWORKS IN A STATIC ASSEMBLY OF DISCS

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Abstract. We present calculations of forces in a static two dimensional sandpile model. The model is very simple supposing spherical, identical, rigid particles on a regular triangular lattice, without friction and with unilateral spring-like contacts. We use a symbolic calculation software to get exact results for several different orientations of the lattice and for different types of supporting surfaces. Special attention is given to the stress tensor and pressure on the bottom of the pile due to their importance in recent works.

1. Introduction

Many numerical simulation techniques are used in the studies of granulates and the extensive use of these techniques gives us many clues that would have been difficult to obtain by other means. But these advantages of numerical simulations are accompanied by many difficult issues. The non linear interaction between two grains makes slow the convergence of the usual algorithms; event driven simulations are unpractical in static situations and forces are hard to define in cases when the grains are considered rigid. Granular systems are also very sensitive to small perturbations as a result of which cumulative roundoff errors may give rise to changes of huge amplitude, making the results unreliable.

The use of numerical techniques is generally imposed by the large number of constituents of the system and not by the complexity of the equations for each grain. In such a case one can use the services of a computer in a different way; using it with a symbolic calculation software as an analytical

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calculator capable of solving huge equation systems as if it were done by a human, but in a much shorter time.

Since the calculation is done without any floating point approximation and with no roundoff errors of any kind, the results obtained are completely reliable. On the other hand, since the implementation is much more complex than usual numerical techniques and since this kind of software is generally unavailable on platforms like Cray, the sizes that can be calculated within reasonable computer resources are much smaller.

2. What model?

We must use a model with some supplementary assumptions that will simplify the equations to a point where the resolution is reduced to solving a (big) linear equation system. To acheive that we place ourselves in the following situation:

1. Discs are identical in all properties; they have the same weight w and radius r. This is not a limitation of the algorithm but a choice made to ease the interpretation.
2. Contacts between discs are elastic and unilateral i.e. when neighboring discs overlap they are repulsed with a force proportional to their overlap. On the other hand when the discs do not overlap no forces are exchanged (dry granular media).
3. The discs are supposed to be stiff; the softness $\tau << r/w$ so that the overlapping of discs is always infinitesimal. In the following we are going to take always the limit $\tau \to 0$. However finite but small softness may also be considered in which case the results would be first order approximations.

Under these conditions the equation system is almost linearized, the only non linear terms are the Heavisides' Θ functions that reflect the unilaterality of the contacts and cannot be linearized near zero.

Dealing with this non linearity is the hardest part of the resolution since no straight forward resolution method for this type of system exists[1]. We have elaborated a trial and error algorithm that looks iteratively for a solution in which the contact network compatible with all of the Θ functions. The flow chart of this algorithm is presented in figure 1. This algorithm provides us the contact network and the forces that are solution of the complete equation system which allow us to calculate some interesting quantities like the pressure profile under the pile (looking for the famous dip) or the stress tensor.

[1]Of course other than simply try all the possible combinations of active/inactive contacts.

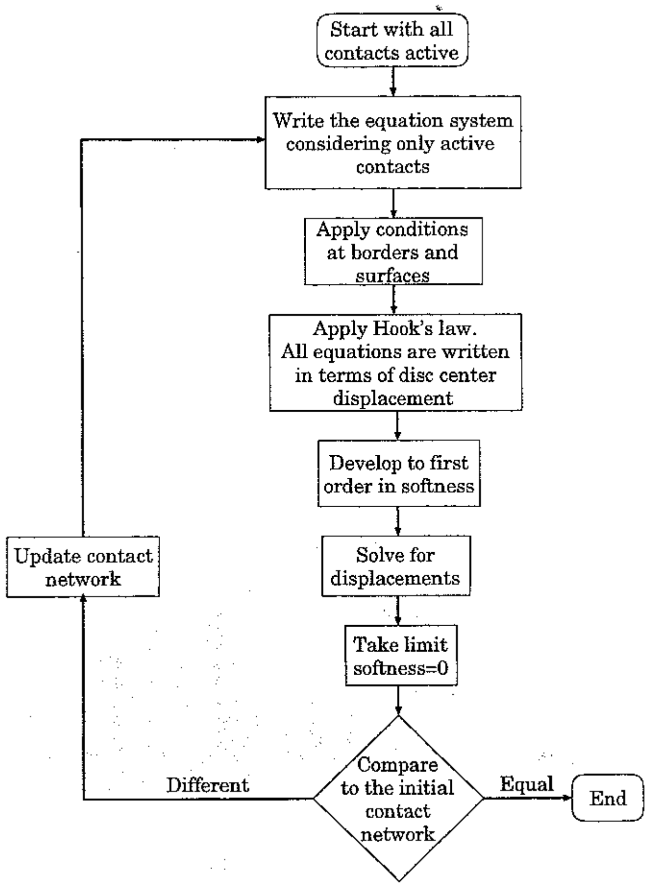
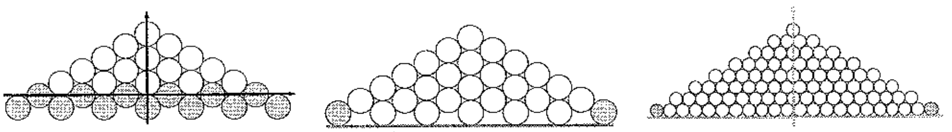


Figure 1. The flow chart of the iterative trial and error algorithm.

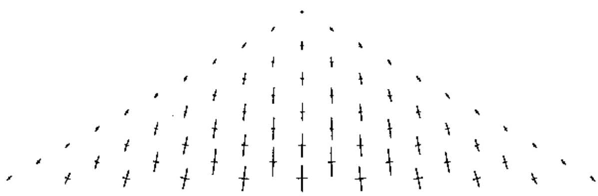


(a) "Tilted" lattice pile on a bumpy floor.

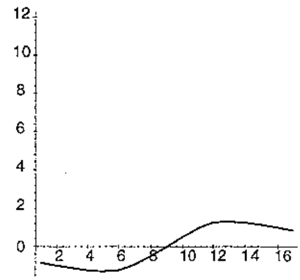
(b) "Tilted" lattice pile on a smooth surface.

(c) "Untilted" lattice pile with 30° base angle.

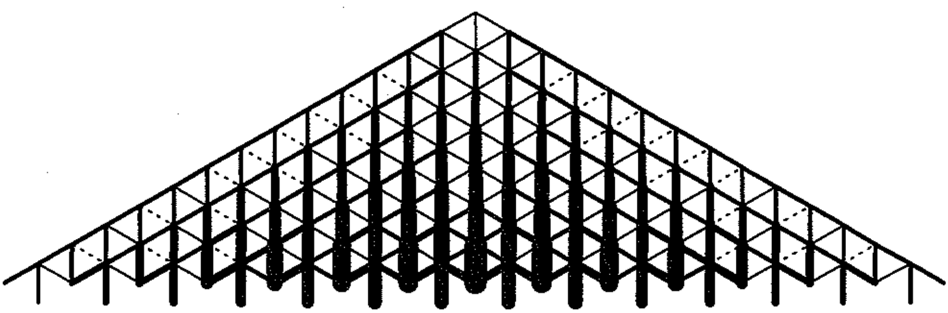
Figure 2. Some of the different configuration studied. The gray discs are discs which centers are fixed to the lattice position (corner stones).



(a) The resulting stress tensor. 11 layers pile.



(b) Force profile on the base for a 14 layers pile.

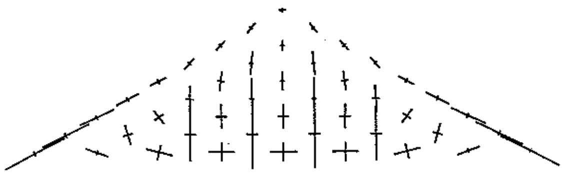


(c) The force network of a 14 layers pile; plain lines width is proportional to the forces, dashed lines represent inactive contacts.

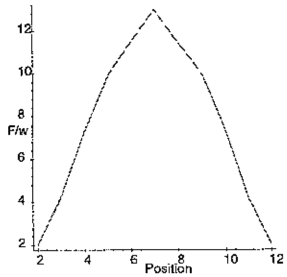
Figure 3. An example of results obtained in case (a). In sub-figure (b) the dashed line is the normal force and the plain line is the shear force

3. Results

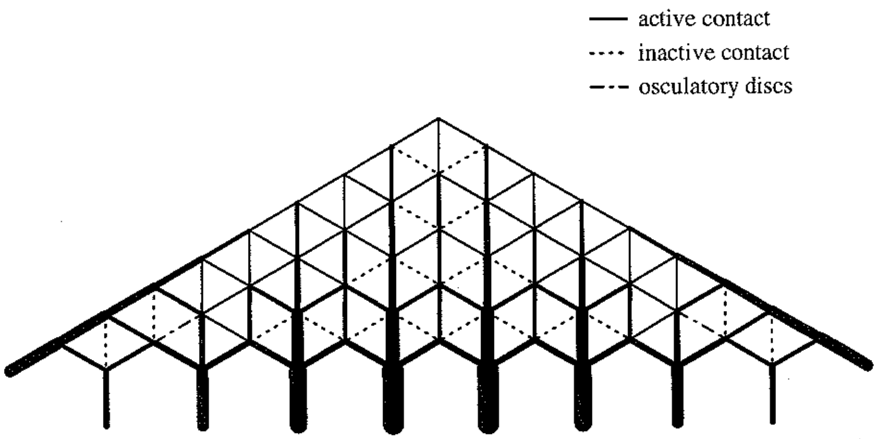
A summary of results obtained in the 3 different configurations shown in figure 2 is found in figures 3,4 and 5. Even though the piles are very similar the results are very different. Small changes like the orientation of the lattice or change of the supporting surface have high impact on all physical characteristics of the pile. While the pressure profile on the base shows a weak dip at the pile's axis in case (c), the other two cases show a hump. In the case (c) we were able to compare with results obtained by [1] using molecular dynamics techniques, and we find a very good agreement. The versatility of the algorithm has also permitted us to calculate the effect of



(a) The resulting stress tensor.



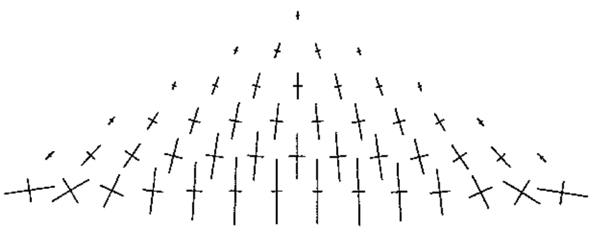
(b) Pressure profile on the base.



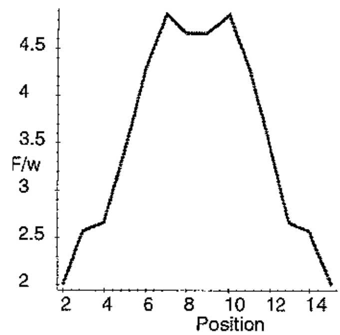
(c) The force network; plain lines width is proportional to the forces.

Figure 4. An example of results obtained in the (b) case with a pile of 10 layers.

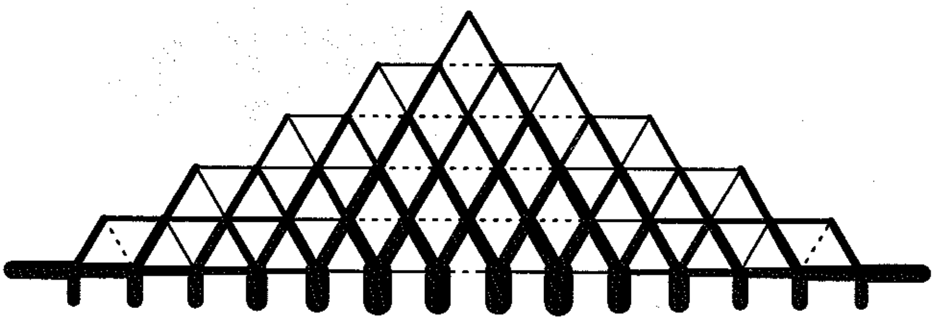
applying external forces to the pile. We confirm, in case (c) the numerical simulations by [1] that shows an accentuation of the dip when applying a force on the corner stones. A discussion of these results can be found in [2].



(a) The resulting stress tensor.



(b) Pressure profile on the base.



(c) The force network; plain lines width is proportional to the forces, dashed lines represent inactive contacts.

Figure 5. An example of results obtained in the case (b) with a pile of 10 layers.

References

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