Intermittency and self-similarity in granular media

H.J. Herrmann
Institute for Computer Applications I, Stuttgart, Germany

ABSTRACT: Granular media show interesting intermittent behaviour in avalanching, pipe flow and the motion inside shear bands. These cases are investigated using theoretical modeling, molecular dynamics, cellular automata and analogies to traffic. Some experimental verifications are discussed. Self-similar structures appear leading to fractal networks of shear bands and the calculation of the tail of a sandpile.

1. INTRODUCTION

Granular media often show intermittent motion in cases where normal fluids flow uniformly. Examples are avalanches along the shape of a heap (Held et al. 1990, Frette et al. 1996), density waves in pipe and hopper flow (Horikawa et al. 1995, Baxter et al. 1989, Medina et al. 1996) and the acoustic emission under shear deformation of dense packings. Intermittent states are in fact known from many situations like turbulence, flierer noise, stick-slip friction, earthquakes and even stock market prices. Intermittent behaviour typically originates from a "critical" state. Its spontaneous appearance has been termed "self organized criticality" (Bak et al. 1987). A critical state is characterized by a dilatation invariance or self-similarity. It is the aim of this presentation to illustrate granular examples of intermittency and to discuss the critical exponents. Also other cases of self-similarity and marginal stability shall be discussed.

2. SPONTANEOUS DENSITY WAVES IN HOPPER AND PIPE FLOW

In 1974 Schick and Verveen published a paper in Nature (Schick et al. 1974) where they measured 1/ν noise in the power spectrum of the density fluctuations of outflowing grains at the outlet of a hopper. They also related this phenomenon to traffic on highways. Later more careful experiments were performed with X-rays (Baxter et al. 1989) in which waves were detected that moved up for hoppers having large opening angle and down if the angle was small. Density fluctuations were also measured using light transmission (Medina et al. 1996) and they found power-spectrum having a power law of exponent 1.32. Matsushita (Horikawa et al. 1995) measured the power-spectrum of the density fluctuations in pipes and found power-law behaviour with an exponent that varied with the vertical position on the pipe. The effects of air played some role in these later experiments.

Air stabilizes the fluctuations into clogs of a characteristic size and velocity. Such transitions from intermittent behaviour to a characteristic wavelength and frequency are also known from the onset of turbulence in fluid mechanics. Pneumatic instabilities as they are also called, appear in various technological applications ranging from gas fluidization to pneumatic transport. Careful experiments have recently been undertaken by T. Raafat (Raafat et al. 1996) on 200 μm glass beads falling in air through a vertical pipe. Several regimes of instabilities were observed with and without clogs of well-defined length and with either constant or oscillating velocity. A system of four coupled equations of motion including the pressure in the air chambers and the permeability of air in the clog was solved and the experimental measurements, which included the pressure, the flux, the length and frequency of clogs and the local density of glass beads, could be reproduced within several percent accuracy.

The intermittent density fluctuations without air, e.g. with large beads, are theoretically more
difficult to explain. Molecular dynamic simulations were performed by Pöschel (Pöschel 1994) of spheres of equal radius flowing in a pipe under the action of gravity. Grain-grain collisions were modelled using the equations of Cundall & Strack (Cundall & Strack 1979). The walls of the pipe were made of randomly placed particles of smaller size. Starting with a random initial configuration of homogeneous density, one observes (after already several tens of seconds) big clusters of particles and also density waves of given velocity. This simulation in vacuum indicates that the inelastic collisions are responsible for the observed phenomena.

To make this point more clear a lattice gas cellular automaton was implemented (Peng & Herrmann 1994, Peng & Herrmann 1995) in which inelastic collisions were added to the “FHP rules”.

This dissipative lattice gas applied on the situation of pipe flow under gravity reproduces the above scenario of density waves and strong clustering fluctuations. The power spectrum gives a power-law with power 4/3 and a peak at the frequency of waves of fixed velocity coming from the periodic boundaries.

Cars on highways can also be considered as a one-dimensional dissipative fluid. Since 1955 density waves are known as kinematic waves to be solutions of the Lighthill-Whitham equation (Lighthill & Whitham 1955) for traffic. Some models to simulate car motion resemble molecular dynamics simulations of granular media having different repulsion forces and other models are more like cellular automata. For one of these models (Nagel & Herrmann) the power spectrum of the density-fluctuations was measured and also a power-law with exponent 4/3 was observed with a peak of the frequency of the kinematic waves.

One open question is the origin of the value 4/3 of the exponent of the power-spectrum. Beredskii (Beredskii 1994) made some speculations using the Obukhov-Kolmogoroff model for turbulence to obtain 4/3 but the calculation of this number from a microscopic model or a complete scaling argument is still missing. Also do experiments up to now not confirm the universality of the exponent (Veje & Dimon 1996, Horikawa et al. 1995) and find in some case more complex behaviour. Arching at the walls may cause additional effects. The intermittent behaviour of density fluctuations might well originate the unpredictability of silo quakes since peaks in force are related to high densities (Ristow & Herrmann 1995, Ristow & Herrmann 1994).

3. FLOW DOWN HEAPS

The angle of repose $\theta$ is a weak concept since its value depends on the procedure how the heap was made and on how the material was prepared. Very recently new experiments were made showing also memory effects due to texture (Grasselli & Herrmann). Real heaps are in fact not perfectly flat but have a tail on the bottom and a rounding on the top. The shape of the tail has been measured in quasi two-dimensional cells for polenta, sugar and lead beads and found to be logarithmic of the form (Alonso & Herrmann 1996):

$$x = \frac{h_m - h}{\gamma} + l \log \frac{h_m}{h}$$

(1)

with $\gamma = \tan \theta$, $h_m$ the height of the pile, $x$ and $h$ the horizontal and vertical coordinate of the surface and $l \approx 3d$ seems a universal constant where $d$ is the grain diameter. A simple model of kinks on which down falling particles can aggregate can be solved (Alonso & Herrmann 1996) using the experimentally observed self-similarity of heaps and gives eq. (1). $l^{-1}$ in eq. (1) corresponds to the aggregation probability of one kink. Applying the same dissipative lattice gas used by Peng (Peng & Herrmann 1995) for density waves one also finds the logarithmic tail of eq. (1) with $l = 3d$.

Figure 1: Digitized image of a heap of the polenta. The diameter of the grains is about 0.5 mm. The height of the heap is 16 cm. Different grey levels show the pile at different stages of growth. The superposed continuous lines in both figures are fits obtained from eq. 1 by taking the values $\gamma = 0.98$ and $l = 1.5$ mm (from Alonso & Herrmann 1996).

*It has been shown using the Chapman-Enskog scheme (Prisch et al. 1986) that the FHP collision rules provide the Navier-Stokes equations and reproduce quantitatively the flow of Newtonian fluids.
The apparent universality of $l = 3d$ is even more surprising if one considers that no direct calculation of the angle of repose from the microscopic properties of the grains like the restitution coefficient or their size or shape has yet been possible, although efforts are under way (Alonso et al). Interesting models have also recently been proposed to explain the stratification of grains of varying size and surface roughness (Makse et al).

In the simple picture put forward by Bagnolds of two angles one of repose $\phi$, and one of marginal stability $\theta$, one expects periodic avalanches of a size proportional to the size of the heap and this is indeed observed experimentally (Liu et al. 1991). For very small heaps (basis less than $N$ grains, $N = \cos \theta_0 / \tan(\theta_0 - \theta_0)$) one observes, however, intermittent avalanches (Held et al. 1990) with a power-law distribution in their sizes. In the case of anisotropic grains (long rice) such a power-law is even obtained for heaps of arbitrary size up to 2 meters (Brette et al. 1996). For more detailed discussions on this question see (Feder 1999).

To describe this intermittency in avalanching which in real life originates the unpredictability of these events, several cellular automata have been proposed. The most famous one in the height model introduced by Bak Tang and Wiesenfeld (Alonso & Herrmann 1996) as paradigm of self-organized criticality. Essential ingredient of these models is a local threshold, like a local angle of repose, beyond which grains are discharged. The eventual chain reactions generate avalanches of all sizes with a power law distribution. The exponent of the distribution depends on the local discharge rules although there are classes of rules having the same exponents.

Although intermittent avalanching is by now well-established, the dependence on the grain properties (restitution coefficient, shape etc.) remain open. It might be that the power-law observed for rice piles is not due to threshold dynamics and chain reactions but is rather related to the surface roughness generated by the vertical stacking of rice grains.

4. SHEAR BANDS

Let us now consider a dense packing (above Reynolds density) of grains in a shear cell. Imposing a normal pressure and a shear velocity one will measure a dilatancy and strong fluctuations in the required shear force. In the generic case (friction angle $\psi \neq$ dilation angle $\phi$) the deformation will localize along shear planes. This process has been simulated using a discrete element method (Tillemans & Herrmann 1995). The grains were arbitrarily shaped convex polygons, the repulsion forces given by Poisson elasticity and periodic boundaries were used. The rotation and energy of the grains were monitored. Fig. 2 shows a shear band characterized by those grains that have turned more than $3^\circ$. The grains move in fact more burst-like reminding stick-slip in friction experiments. The histogram of local energies follows a power-law with exponent $-1$. This coincides with the Gutenberg-Richter law known for the energy histogram of real earthquakes. This might not be surprising if one considers that the St. Andreas fault is essentially a tectonic shear cell. Cohesion seems to play a secondary role in the sense that it does not modify the exponent characterizing the intermittency. Analogously to seismographs used to detect earthquakes one would expect acoustic emission to be an interesting tool to investigate shear cells. A systematic study of intermittent acoustic emission analogous to the one done on rocks (Petri et al. 1994) and volcanos (Diodati et al. 1991) have not yet been undertaken to my knowledge.

A large scale calculation of sheared granular soil using an explicit Lagrangian solution of non-associate ($\phi \neq \psi$) Mohr-Coulomb plasticity has shown (Pollakov & Herrmann, 1994) that shear-bands form fractal networks. Changing the scale for instance by increasing resolution. Space is covered by an ever increasing number of shear bands of diminishing width. The lower cutoff which would be given by the grain size is not included in the continuum equations which were solved here and effectively replaced by the numerical grid. The fractal dimensions obtained numerically agree with those measured from digitized photographs of shear bands on granite from the Pyrenees (Pollakov & Herrmann 1994).

![Figure 2: Sheared system of polygonal cells. The grey cells have rotated more than 3\(^\circ\).](image-url)
5. OUTLOOK

If modern statistical physics can contribute to the understanding of granular media the study of intermittency is one example. Generic laws similar to Kolmogoroff scaling in turbulence are valid and their characteristic exponents universal, i.e. independent on the properties of the grains, at least as long as the grains fall inside a certain category. Histograms and power-spectra are useful to predict largest events or characteristic frequencies from a theoretical basis seems a useful information. There also exists a profound relation between intermittent behaviour and self-similar structures explaining some of the spatial patterns observed.

REFERENCES

Grasselli Y. & H.J. Herrmann, preprint.