On the angles of dry granular heaps

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On the angles of dry granular heaps

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Abstract

We perform careful experiments in a turnable cell to determine the angle of repose and various other angles of dry granular materials. We study the dependence of the angle on the presence of walls and find an exponential decay on the distance between the walls. Modification of the granular fabric close to the surface changes the static angle of repose and the effect can still be noted after several inclinations of the cell indicating the existence of a memory effect. The dynamic angle of repose is particularly small after the first inclination of the cell. When part of the heap is removed through an outlet, three different angles are obtained showing the effect of fabric and heap preparation on the dynamic angle.

1. Introduction

The angle of repose $\theta$ of a heap of dry granular media is an ancient concept and an everyday observation on sandpiles, coal heaps, moraines etc. Its value is used in several continuum theories like avalanche models [1–3], stratification models [4] or the plasticity theories used in soil mechanics [5]. Bagnolds [6] noticed the existence of a second angle: the “angle of maximal stability” $\theta_s$ several degrees larger than the angle of repose which is the maximal angle that a granular packing can attain when carefully tilted. Beyond this angle the surface becomes unstable and avalanches appear. This second angle is, however, rarely taken into account in the theoretical approaches. Besides the evident fact that $\theta$ depends on the shape and size distribution of the grains, experiments have shown that the angle of repose increases with density [7]. The data suggest a law of the form $\tan \theta \propto a (\rho - \rho_m) + b$ where $\rho_m$ is the maximal attainable density and $a$ and $b$ are constants. Many other experimental studies have noticed that $\theta$ also depends on humidity, packing history and boundary conditions but no systematic study of these effects has yet been undertaken to our knowledge.

The aim of the present work is to better understand the rules governing the formation of a given heap angle by doing very careful experiments and by measuring the angles

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with precision. In particular we investigated the dependence on the presence of walls, on the formation history and on texture. We reveal a rather complex picture.

In the next section we describe the experimental setup. In the following three sections we present these experimental results. In the last section we will discuss the results and try to give a general physical picture about the angle formation.

2. Experimental setup

Experiments have been performed using a cell composed of two parallel glass plates (cf. Fig. 1). Glass was chosen to avoid undesirable electrostatic effects on the granular materials due to friction when we fill the cell. The glass plates can be separated by different spacers in order to vary the cell thickness, \( d \), between 1 and 10 mm by steps of 1 mm. These configurations allows us to investigate the effects of rigid walls on the angle of repose. The cell is mounted on a tilting device which permits to determine the difference between the static and dynamic angles for a given material. The cell is also equipped with a slit situated at its bottom that was used to conduct the outflow experiments (described later).

We have worked with two different kinds of granular media in our experiments: powders (sand and glass splinters) and glass spheres. For each one, we had different possible particle sizes ranging from 80 to 350 \( \mu \)m for powders and from 100 to 400 \( \mu \)m...
for glass spheres. All experiments presented below have been performed as well with powders as with spheres in order to get a comparison between their behaviour. Moreover, in order to get experimental results with a rather high precision, each experiment described in this paper has been repeated at least ten times to obtain an average value with a reasonably small statistical error bar. The main source of errors comes from reading off the millimetric scale. So we have worked on large piles to minimize this effect.

The first step of this study was to develop the appropriate way to fill the cell. In fact, one could believe that this point does not have a great influence on the results, but we found that if no precaution is taken on the filling procedure of the cell, experiments were not precisely reproducible and we got larger errors. To illustrate this effect, we have realised experiments filling the cell slowly or rapidly and we measured the corresponding angle of repose for a given thickness. After averaging several experiments for the two cases, we get nearly the same mean value for the angle of repose. The difference between the slow and rapid situations was only $0.3^\circ$. But the corresponding errors were extremely different: the error bars obtained in the rapid filling process of the cell was four times larger compared to the error of the slow process. We found that slowly filling the cell allowed us to access experimental results with a good reproducibility and acceptable errors.

Humidity also plays an important role in our experiments. That is why for all experiments, it has been monitored in order to compare only results with the same degree of humidity. We also monitored the temperature but its role seems minor.

3. Angle of repose and wall effects

It is clear that the angle of repose, $\theta$, of a given granular material is modified by the presence of rigid plates. In this case, the angle is always larger than the one of a totally free pile mostly because of arching phenomena between the two plates. By free pile, we mean the case of a real three dimensional pile i.e. without the presence of any walls. In order to determine the behaviour of the angle of repose as a function of the cell thickness, $d$, we measured this angle for several granular media (powders and glass spheres) at different cell thicknesses. For each material, we averaged over several measurements at constant humidity and temperature. The dependence of $\theta$ as a function of $d$ can be determined by plotting the normalised ratio $(\theta(d) - \theta_\infty) / \theta_\infty$ as a function of $d/a$ where $\theta_\infty$ and $\theta(d)$ are respectively the angle of repose of the free pile and the angle obtained in a cell of thickness $d$ with $a$ being the average particle size. At this point we need to know the value of $\theta_\infty$ for each material tested. This quantity can be measured in a simple way. We used a cubic cell with a wall separation of 75 mm (cf. Fig. 2). The granular material is placed on a plate on the top of the cell and the plate is pulled a little bit in order to let the grains fall down into the cell. The material then flows along one wall of the cell and the pouring process of the material is therefore the same as in the case of our two dimensional cell. This cell is also
Fig. 2. Cubic cell used to determine the angle of repose of a material without wall effects. The upper plate is pulled a little to let the material pour into the cell. With the ratio of the wall separation to the particle diameter larger than 180, we have assumed that the angle measured in this way is indistinguishable from the one of a real three dimensional pile.

equipped with an outlet which will be useful to conduct outflow experiments at such cell thickness. We think that at such values of $d$, which give for all materials used a ratio $d/a$ greater than 180, wall effects do not arise and that the obtained angle is undistinguishable from the one of the free pile. To confirm this assumption, we have also constructed a free standing three dimensional pile on a table and measured its angle of repose. The difference obtained between the free situation and our cubic cell was less than 0.3° which is within the statistical error bar.

We have plotted, in Fig. 3, the ratio $(\theta(d) - \theta_\infty)/\theta_\infty$ versus $d/a$ and also versus $d$ for results obtained on different powders and glass spheres having an average particle size $a$ and with nearly the same humidity of 20%. We found that the behaviour of the angle of repose as a function of the cell thickness is governed by an exponential law of the form $\theta(d) = \theta_\infty (1 - e^{-d/d^*})$ having a characteristic length $d^*$ which depends on the granular material used: this length is simply given by the slope of the straight line in Fig. 3. Let us now describe this behaviour as a function of $d$. What is surprising is that we can obtain a scaling law for all grains sizes, and of course for the same kind of materials, if we express $(\theta(d) - \theta_\infty)/\theta_\infty$ versus $d$ only. For example, on Fig. 3b, we note that the values are rather the same for particle size of 180 and 250 $\mu$m, which are media made of sand, but different from the ones obtained with a particle size of 80 $\mu$m (glass splinters). The difference comes from the values of $\theta_\infty$ which depends on the nature of the granular medium. Nevertheless, the slopes of the curves are nearly the same. On the contrary (Fig. 3a), for glass spheres media, we get the same $\theta_\infty$ for all sizes; thus all experimental points for the three sizes used collapse on a single curve.

Results retrieved from these curves are presented in Table 1 expressed in units of particle sizes $a$. We also give the rate of humidity corresponding to each experiment. The characteristic length $d^*$, representing a distance over which the confining walls have an effect on the angle of repose, could only be dependent on $a$ and on the humidity. We found that $d^*$ increases as the particle size decreases which is the opposite to
Fig. 3. Semi log plot of the normalised ratio \((\theta(d) - \theta_\infty)/\theta_\infty\) as a function of \(d/a\) and \(d\), of a material having an average particle size \(a\), \(d\) being the cell thickness. \(\theta_\infty\) is the angle of repose of a three dimensional pile (i.e. without wall effects) which depends on the nature of the granular medium. This behaviour is governed by an exponential law with a characteristic length which is material dependent. (a) Glass spheres (humidity of 30%). (b) Powder: sand \((a = 180 \mu m \text{ and } a = 250 \mu m)\) and splinters of glass \((a = 80 \mu m)\) (humidity of 20%).
the expected behaviour. This may be an effect of humidity. In fact, if wall effects on the angle of repose are interpreted through arching, we can assume that capillary forces arising between particles become more important as the particle size decreases. Consistent with this explanation, we have observed by repeating the same experiment with the same materials that lower values of humidity give smaller characteristic lengths (cf. Table 1).

4. Memory effects and internal structure

The following experimental procedure is applied next: for a given cell thickness, we fill the cell with a granular medium (powder or glass sphere) as in the previous section. We measure the first dynamic angle, $\theta_1$, resulting from the filling. The cell is then tilted by an angle $\Delta \theta_1$ until a first avalanche occurs. The first static angle is simply obtained by adding $\theta_1$ and $\Delta \theta_1$. The avalanche gives rise to a new dynamic angle $\theta_2$ which we also measure. This procedure is repeated several times to get successive values of dynamic and static angles and is similar to a step by step decomposition of the rotating drum experiment.

For a specific material and a given thickness $d$, different experiments have been realised. Close to the surface we changed the internal structure of the packing and eventually its density by passing a rigid rod parallel to the surface of the pile at several depths. Fig. 4 presents the results obtained for powders on dynamic angles and the difference between dynamic and static angles for a cell thickness of 3 mm and an average particle size of 350 $\mu$m. Larger thicknesses of the cell do not change the general form of the plots but just lower all the values by a small amount. Several modifications of the initial pile have been realised: the rigid rod was passed below the surface of the pile 1 mm, 1 cm and 3 cm deep inside. The first observation we make is that, for successive tiltings, the dynamic angle is unaffected by the modifications. But, after the first avalanche occurs, we get a sharp decrease of the dynamic angle which then tends to a rather constant value. This effect, which exists for all granular materials used in our experiments, may be related to the construction process of the pile. The first dynamic angle is obtained by filling the cell: grains are injected at the top of the pile and then roll down along the free surface of the pile. The second, and all the other dynamic angles (obtained after successive tiltings) are generated by an avalanche which
Fig. 4. Experiments on the memory effect of a pile realised on a powder with an average size of 350 μm and a cell thickness of 3 mm. (a) Dynamic angles obtained as the experimental tilting procedure is repeated (see text). The dynamic angle is unaffected by modifications of the internal structure made at several depths through the pile. (b) Corresponding values of the difference between dynamic and static angles obtained for the same experiment. The static angle is drastically affected by the modifications and we can also observe a memory effect after several successive tiltings (i.e. after several avalanches). The various data are slightly shifted in relation to each other in horizontal direction to make them more distinguishable.

comes from avalanches of all grains on the free surface. We notice that as long as the experimental procedure is repeated, the process of successive dynamic angle formation tends to a constant angle. We can suppose that when we fill the cell, we create a given internal structure (which is changed by the rod if sufficiently deeply pushed in) which is characteristic of the filling process. After the first avalanche, the internal structure near the surface is changed because it comes from a collective motion of all the grains close to the free surface of the pile. This can explain that we get a lower dynamic angle. This new internal structure is half way between the initial one and the structure formed by others successive avalanches. After several tiltings, however, the memory of the initial structure is lost and the dynamic angle of repose becomes constant. The new internal structure is then independent of the initial one.
The static angle is, on the contrary, drastically affected by the modifications induced by the rod on the initial pile. We can see that for a small surface modification (1 mm deep), we obtain the same static angle as without modification. By passing the rigid rod through the pile at a sufficient depth, the internal structure and density are affected which give an increase in the static angles. We can clearly observe an interesting memory effect as successive avalanches occur. Only after having repeated the tilting three times (i.e. after three avalanches), the pile has totally forgotten its original structure: the difference between dynamic and static angles becomes the same for all initial modifications.

Similar experiments have been performed with glass spheres. The same experimental procedure was used to check the history dependence. Unfortunately, it was not possible to obtain precise results because of the sensitivity to vibrations. It seems however that there is no memory effect in granular media composed of glass spheres: even for a deep modification of the internal structure, we get no difference in the static angles. In general glass spheres are quite sensitive to external vibrations (more than powders) and obtaining precise results is difficult. Nevertheless, we will try to make this kind of experiments on glass spheres with more accuracy to confirm the preceding statements.

5. Outflow experiments

Another study we can perform with our cell are outflow experiments with the help of the hole situated at the bottom of the cell. The principle of this experiment is to fill first the cell with a given granular material and then to open the hole in order to evacuate a part of the pile. The rate of the particle flow through the hole can be adjusted by varying the width of the slit in order to impose slow flows which make the experiments more reproducible. At the end of the experiment, we find two new angles \( \theta_a \) and \( \theta_b \) (cf. Fig. 5) that we can measure and compare to the initial angle of repose \( \theta \). Experiments have been performed on powders and glass spheres for several cell thicknesses. Results for the three angles are presented in Fig. 6 for a powder having an average size of 250 \( \mu m \) and for glass spheres with an average size of 112 \( \mu m \). We notice that the behaviour of powders is again different from that of glass spheres. For the latter, the three angles are nearly identical as opposed to powders where, for all the thicknesses used, these angles are different and always follow the order \( \theta_a > \theta_b > \theta \). A similar experiment has been realised with our experimental cell but with a hole situated in the left lower corner of the cell. Of course, in this experiment, we just created one angle \( \theta_a \). Again we found that \( \theta_a \) was greater than \( \theta \) and that the difference between them was the same as the one obtained with the hole placed like in Fig. 5.

Let us now consider the case of powders. We have to investigate whether the differences observed between these three angles still occurs when no wall effect is present. So, we have performed outflow experiments using the cubic cell shown on Fig. 2. The wall separation is now 75 mm which we assume to be sufficient to neglect wall effects
on angles. The results we have obtained for the powder with a grain size of 250 μm are: $\theta_{\infty} = 32.2 \pm 0.4$, $\theta_a = 36.8 \pm 1.6$ and $\theta_b = 35.4 \pm 1.5$. Error obtained on the asymptotic values of $\theta_a$ and $\theta_b$ are quite large and we were unable to reduce them even with high statistics mostly because of the small size of the new heaps created by outflow; thus, reading errors on these angles are larger compared to results obtained with our experimental cell. Nevertheless, again we get $\theta_a$ and $\theta_b$ greater than the initial angle $\theta$. We cannot say that $\theta_a$ is greater than $\theta_b$ because the errors bars overlap, but in all experiments, we found $\theta_b < \theta_a$. In Fig. 6a, we have also plotted the best exponential fit which was possible to obtain for the three sets of experimental points, $\theta$, $\theta_a$ and $\theta_b$ according to the exponential law found in Section 3. The results allow us to think that the differences found between the angles is not only a wall effect but a consequence of an internal structure.

When we construct a pile by pouring particles from the top, we create an average grain orientation which is nearly parallel to the surface of the pile. Outflows modify this relative orientation of the angle $\theta_a$: they generate a structure more or less perpendicular to the surface and this can explain that $\theta_a$ is always the larger angle. For $\theta_b$, the internal structure is similar to the initial one; here only the formation process of the angle is different. The different behaviour observed for glass spheres could be related to the isotropic shape of the particles.

Our observed angles $\theta_a$ and $\theta_b$ strongly differ from the values predicted for these angles by the Fixed Principal Axis (FPA) theory of Wittmer et al. [9,10]. The only difference in the experimental procedure seems to be that Wittmer et al. want to remove the sand “grain by grain” while we do a very careful decomposition. Since our outflow is very slow and the velocity profile of the outflowing sand only becomes non zero close to the stagnation zone near the end of the outflow experiment, we believe that our experimental conditions are not far away from Wittmer et al.’s proposed conditions. We therefore believe that some of the conditions of Wittmer et al., like the use of the usual stability criterion for the case of decomposition might not be valid.
Fig. 6. Dependence of the angles created by the outflow experiments (cf. Fig. 5) as a function of the cell thickness. (a) Powder with an average particle size of 250μm. (b) Glass spheres with an average particle diameter of 112μm.
6. Discussion and conclusion

The presence of walls can considerably increase the angle of the heap (typically 10–20%) which evidently comes from the fact that force lines from the packing are supported by the wall making bridges and giving more stability. Let us propose a simple model. The probability $p_n$ of forming a bridge of $n$ grains can be estimated by assuming a random assembly, giving $p_n \propto p^n$, where $p$ is the probability of having a supporting grain in the direction of the line. The fraction of weight supported by the wall should therefore decay like $e^{(d/a)\ln p}$ with the distance $d$ from the wall where $a$ is the grain diameter (note that $d = na$ and $\ln p < 0$). One might assume that the excess angle is proportional to this extra support of the weight by the wall. We confirmed the exponential decay but not the dependence on $a$.

We cannot explain why in our experiments no evidence for a linear dependence of the characteristic length of the exponential decay is found as a function of grain diameter. It could be that this is due to the relative polydispersity of our granular material giving rise to a more complex dependence of the probability $p$ on the precise distribution of grain sizes and shapes. It could also be that the above picture of random force lines is wrong and that the topology of the force network depends on grain size. This would be the case if, due to humidity, capillary forces act which would be consistent with our experimentally observed dependence on humidity.

Concerning the formation history of a granular shape, our observations led us to distinguish five (and not two) fundamentally different types of angles depending on the formation procedure used. A similar division was already proposed by Brown and Richards [8]. We list the five types in decreasing value of the angle.

1. The static angle $\theta_S$ which is obtained by carefully tilting a given packing to its stability limit.

2. The outflow angle $\theta_O$ obtained by carefully decomposing an existing heap through very slow outflows. This corresponds to decomposing a pile from top to bottom by removing the grains very gently since the outflow velocity at the remaining stagnation surface is zero.

3. The heap angle $\theta_H$ obtained by carefully pouring grains on the top of the heap (our initial configuration). In this case the growth of the pile is characterized by the formation of kinks and an oriented texture in the packing giving rise to an orientation of the force lines as described by the FPA model [9,10]. Kink-models like the one of Ref. [11] should apply to obtain $\theta_H$ from the properties of individual grains.

4. The drum angle $\theta_D$ observed in the rotating drum. Here no kinks appear and no texture is observed. Arguments based on dilatancy seem appropriate to calculate this angle.

5. The avalanche angle $\theta_A$ observed after a big avalanche. This avalanche could have been triggered by a shock but also due to the breakdown of a particularly consolidated surface as it is the case at the first tilting in Fig. 4a. In this case inertia prevents the grains from being stopped by the asperities of the surface and $\theta_A$ will depend on speed and size of the avalanche.
This does not mean that there are just discrete possible angles, since each of the five types can attain a continuum of values depending on density, humidity, vibrations, etc. Experimentally, one finds
\[ \theta_S > \theta_O > \theta_H > \theta_D > \theta_A \]

In Fig. 4a, we see subsequently \( \theta_S, \theta_A \) and \( \theta_D \).

The decomposition angles \( \theta_S, \theta_O \) and \( \theta_A \) depend on the bulk properties (structure, stress transmission, density) of the packing they decompose. In Fig. 6a, we see how \( \theta_O \) depends on the texture; while for \( \theta_b \) decomposition was along the layering orientation (direction of kink motions) for \( \theta_a \), it was roughly orthogonal to the layers. We found \( \theta_a > \theta_b \). Another example is the dependence of \( \theta_S \) on the modification with the rod in Fig. 4b. It is likely that the main effect of the rod consists in increasing (slightly) the density at least if it is pushed in sufficiently deep. In that case our results confirm the observations of Ref. [7]. Interestingly the surface itself does not seem very relevant since the rod penetrating only 1 mm has no noticeable impact on \( \theta_S \).

The striking difference in the behaviour of spheres and non-spherical powders is not unexpected. The physical differences may be that frustration of grain motion is static for powders but dynamic for spheres [12]: Spheres can roll and relax faster towards dense packings, while in powders with many shapes large holes can be stable just because no close by grain has the right shape to fill it.

**References**

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