Frustration and slow dynamics of granular packings

Mario Nicodemi, 1,2 Antonio Coniglio, 1,2 and Hans J. Herrmann 1,3

1PMMH ESPCI, 10 rue Vauquelin, 75231 Paris Cedex 05, France
2Dipartimento di Scienze Fisiche, Universita di Napoli “Federico II,’’ INFN and INFN, Sezione di Napoli, Mostra d’Oltremare, Pad. 19, 80125, Napoli, Italy
3ICA I, Universita Stuttgart, Pfaffenwaldring 27, 70569 Stuttgart, Germany

(Received 4 November 1996)

Dynamical processes in granular media show fascinating behavior [1–4]. The particular role of disorder and fluctuations in granular dynamics has led several authors [1,5–7] to propose an analogy to frustrated statistical systems such as spin glasses [8].

Recently a frustrated lattice gas model has been introduced to describe static and dynamic properties of granular materials [9]. This microscopic model is based on an analogy with frustrated percolation [10], expressed in terms of a Hamiltonian formalism in which disorder and frustration are key elements. In this statistical model, “vibrations” play the role of temperature in usual thermal systems [1,11–13]. Its quenched disorder and the consequent frustration try to describe the general physical mechanisms underlying the phenomenon of “‘geometrical’” frustration known in granular media. The same model, without the gravitational contribution in the Hamiltonian, has been previously related to the physics of the glass transition in glass forming liquids [14].

Here we describe the combined effects of vibrations and gravity in this model, and compare them to experimental data. A well known experiment with granular systems is the compaction of sand. When a box filled with loose packed sand is shaken at low amplitude, density visibly increases [15]. If in addition, the density goes beyond a definite threshold, the mechanical properties of sand abruptly change and the granular structure cannot be sheared any longer without a volume increase. This phenomenon, very important in practical applications [16], was observed by Reynolds [17], and is referred to as the “Reynolds” or “dilatancy” transition. In the present model an analogy appears between the cooperativity effects underlying the Reynolds transition in granular media and the actual spin glass transition where a diverging length naturally exists [7,9].

This paper also analyzes the grain density relaxation in different dynamical situations, and relates our observations to corresponding phenomena in real experiments. We find that in the present model the logarithmic behavior, known from experimental measurements in sequences of taps [15], is recovered, and further predictions are possible. We observe a different dynamical behavior for grain deposition in a single vibration process, where stretched exponentials are found. The robustness to changes of these structural properties is also analyzed.

II. MODEL

The model we study here is described in [9], and we briefly summarize its essential characteristics. It consists of a system of particles diffusing on a square lattice whose bonds are characterized by fixed random numbers $\epsilon_{ij} = \pm 1$ (see Fig. 1). On site $i$ we set $n_i = 1$ if a particle is present and 0 otherwise. Particles are characterized by an internal degree of freedom $S=\pm 1$ and are subjected to the constraint that whenever two ($i$ and $j$) are neighboring, their “spin” must satisfy the relation

$$\epsilon_{ij} S_i S_j = 1,$$

i.e., they have to fit the local “‘geometrical’” structure. At high enough density, particles feel the effects of the “‘quenched’” frustration imposed by the choice of the $\epsilon_{ij}$. In fact, in resemblance to frustrated percolation [10], they can never close a frustrated loop in the lattice because along such loops the quantity $\sum_{i,j \in \text{loop}} (\epsilon_{ij} S_i S_j - 1)$ cannot be zero [8] as imposed by the condition of Eq. (1). In the system there will then unavoidably be empty sites. The bond variables $\epsilon_{ij}$ model the general effects of geometrical frustration in granular systems due to the shapes and arrangements of particles.

FIG. 1. Schematic picture of the lattice model considered here. Wavy and straight lines represent the two different kinds of bonds ($\epsilon_{ij} = \pm 1$). Filled (empty) circles are present particles with spin $S_i = +1$ ($S_i = -1$).
The internal variables $S_i$ describe local quantities, as rotations and positions, which actually determine the geometrical frustration [6,7].

We want to study this system in presence of "external vibrations" and "gravity." So we define a dynamics in our model as a random diffusion of particles on a square lattice tilted by $45^\circ$ (see Fig. 1) in such a way as to preserve the constraints of Eq. (1). The particles attempt a move upward with probability $P_2$ and downward with $P_1$ (with $P_1 + P_2 = 1$). The move is made only if the internal degrees of freedom satisfy Eq. (1). Similarly a spin flips with probability one if there is no violation of Eq. (1), and does not flip otherwise. In the absence of vibrations, the effect of gravity imposes $P_2 = 0$. When vibrations are switched on, $P_2$ becomes finite. The crucial parameter which controls the dynamics and the final density is the ratio $x(t) = P_2(t)/P_1(t)$ which describes the amplitude of the vibration.

This model can be described in terms of the following Hamiltonian in the limit $J \to \infty$ (see [9])

$$-H = \sum_{\langle ij \rangle} J (\epsilon_{ij} S_i S_j - 1) n_i n_j + \mu \sum_i n_i,$$  \hspace{1cm} (2)

where $S_i = \pm 1$ are spin variables, $n_i = 0$ and 1 occupancy variables, and $\epsilon_{ij} = \pm 1$ quenched interactions associated with the bonds of the lattice. Hamiltonian (2) opens the way to a definite correspondence with disordered magnetic systems as spin glasses, and was actually proven to undergo a spin glass transition at high density (or low temperature) [14,18]. Here we just note that it reduces in the $\mu \to \infty$ limit to the usual $\pm J$ Ising spin glass [8], and in the limit $J \to \infty$ to a version of site frustrated percolation [10,14]. When the particle number is fixed, the configuration space of the system obtained in this last limit is the same as that of the frustrated lattice gas introduced at the beginning of this section.

The Monte Carlo simulations of the model described above are performed on a tilted lattice with periodic boundary conditions along the $x$ axis and rigid walls at bottom and top. After fixing the random quenched $\epsilon_{ij}$ on the bonds, a random initial particle configuration is prepared by randomly inserting particles of given spin into the box from its top and then letting them fall down, with the described dynamics ($P_2 = 0$), until the box is filled. The two basic Monte Carlo moves (the spin flip and particle hopping) are done in random order. To obtain an initial low density configuration we do not allow particle spins to flip in this preparation process. The state prepared in this way has a density of about 0.518 which corresponds to a random loose packing particle in two dimensions.

III. SIMULATING VIBRATIONS

As already mentioned, we study the effects of "vibration" by using a finite value for $P_2$. It is experimentally known that sand, randomly poured into a box, reaches higher density states after shaking. Let us explore this phenomenon in our model.

![FIG. 2. Final bulk density $\rho_\tau(\tau)$, in static configurations, as a function of the logarithm duration $\tau$ of the vibration, for a box of size 100×200. A characteristic vibration duration $\tau^* \approx 2 \times 10^2$ appears, below which the final density $\rho_\tau$ is not affected by the vibrations.](image)

A. Density relaxation in a single "tap"

As first we choose to decrease the initial ratio $x_0 = P_2(t=0)/P_1(t=0)$ in time according a linear law as $x(t) = x_0(1-t/\tau) \theta(\tau-t)$ and $t > 0$, where $\tau$ is the duration of the vibration or the inverse velocity of quenching (we take $x_0 = 1$). This process corresponds, in the magnetic analogy, to a quench of the system from a high temperature state. With this procedure the systems attains a final "static" configuration which is defined by the criterion that during a fixed time $t_{\text{repose}}$ nothing changes any longer. In our simulation we fixed $t_{\text{repose}} = 300$ much longer than any intrinsic time in absence of vibration. Time $t$ is measured in such a way that one unit corresponds to one single average update of all particles and all spins of the lattice.

The data we present in this section were obtained in a box of size 100×200, and their states have been averaged over 32–512 different $\{\epsilon_{ij}\}$ configurations (according to the value of $\tau$). These values, as those used below, are chosen to control finite size effects on our Monte Carlo data as much as possible. They are large enough to say that our results are strongly robust to size changes.

After a vibration cycle has been applied as described above, the system presents final densities which clearly depend on the value of the $\tau$ of the vibrations. As depicted in Fig. 2, the final "static" bulk density $\rho_\tau(\tau)$, defined as the mean density in the lower 25% of the box, increases asymptotically with $\tau$, reaching an ideal maximal density value $\rho_m$ when $\tau \to \infty$ (from our data we have roughly $\rho_m \approx 0.79$). From the data in Fig. 2 one sees that a characteristic value of $\tau$ exists, below which vibrations do not affect the final $\rho_\tau$. For our system size this value $\tau^*$ corresponds to about $\tau^* \approx 2 \times 10^2$.

During the dynamical process described above, we have recorded the time dependence of the mean bulk density $\rho(t, \tau)$, measured as the mean density in the lower 25% of the box at time $t$ as depicted in Fig. 3. At $t = 0$ the density starts from the initial low value defined by our preparation rule $\rho_0 = 0.518$. With increasing time, it approaches a definite plateau which corresponds to the "static" limit $\rho_\tau(\tau)$. It is evident that this process takes places on time scales which depend drastically on $\tau$. In contrast to the functional form proposed by other models (see, for instance, those quoted in
They undergo a change of behavior at \( t_i \), which the final value \( \rho_i(\tau) \) reported in Fig. 2. Dashed lines are fits using Eq. (3), and bold continuous lines are stretched exponential fits according to Eq. (5), as explained in the text. Their parameters are respectively reported in Figs. 4 and 6.

[19–21], our data are reasonably well fitted by a Fermi-Dirac function

\[
\rho(t, \tau) = \rho_i(\tau) - \left[ \rho_f(\tau) - \rho_i(\tau) \right] \left( 1 + e^{-t_i/\tau_0} \right) / \left( 1 + e^{(t - t_0)/\tau_0} \right),
\]

where \( \rho_i(\tau) = \lim_{\tau \to \infty} \rho(t, \tau) \). These fits are shown by the dashed line in Fig. 3 (\( \rho_i = 0.518 \) is the initial state density). The two fitting parameters \( \tau_0 \) and \( t_0 \) are reported in Fig. 4. They undergo a change of behavior at \( \tau \sim \tau^* \):

\[
\tau_0(\tau) = \tau_+ \quad \text{if} \quad \tau < \tau^*,
\]

\[
\tau_0(\tau) = (\tau/\tau_2)^n + a, \quad t_0(\tau) = (\tau/\tau_2)^n + b \quad \text{if} \quad \tau > \tau^*,
\]

where \( \tau_+ = 26, \tau_2 = 0.5, \tau_- = 48, \tau_1 = 1.15, \tau_2 = 0.02, \) and \( a = 0.6 (a = -12 \) and \( b = -5 \). \( \tau^* \) is actually defined as the intersection point of the two curves in Eq. (4). The data for the density relaxation may be rescaled according to Eq. (3), and the scaling plot is given in Fig. 5. This picture reveals that, although the data scale well with respect to the variable \( (t - t_0)/\tau_0 \), for long times the simple master function provided by Eq. (3), which implies an exponential relaxation, seems to show a small but systematic deviation from the Monte Carlo (MC) data. These observations lead us to try better fits in this long time region and our data seem well fitted by a "stretched exponential" form

\[
\rho(t, \tau) = \rho_i(\tau) - [\rho_f(\tau) - \rho_i(\tau)] \exp\left[ -((t - t_0)/\tau_0)^\beta \right],
\]

as shown in Fig. 3 with the bold continuous lines. In Eq. (5) \( d_0 \) is an initial arbitrary density (we take \( d_0 = 0.60 \)) above which we actually make the fit. This is a four parameter fit; however, we find that one can take the factor \( f \) equal to \( f = 1.3 \) and the exponent \( \beta = 2.3 \) as independent of \( \tau \). The parameters \( \tau_0 \) and \( t_0 \) of Eq. (5) have moreover nearly the same behavior as a function of \( \tau \) as the ones described above for the parameters of Eq. (3) (see Fig. 6). Also in this case Eq. (4) approximately holds [actually \( \tau_+ = 62, \tau_2 = 0.2, a = -19, \) and \( a = 0.67 \) for \( \tau_0(\tau), \) and \( t_+ = 25, t_1 = 1.6, \tau_2 = 0.0008, b = -56, \) and \( a = 0.42 \) for \( t_0(\tau) \)]. The relaxation

\[
\rho(t, \tau) = \rho_i(\tau) - [\rho_f(\tau) - \rho_i(\tau)] \exp\left[ -((t - t_0)/\tau_0)^\beta \right].
\]

FIG. 3. Bulk density \( \rho(t, \tau) \) relaxation, in a single vibration process (a single "tap"), as a function of log time \( t \) for different values of the vibration duration \( \tau \) (from left to right \( \tau = 1.64 \times 10^3, 1.64 \times 10^2, 6.56 \times 10^2, 1.64 \times 10^3, \) and \( 1.64 \times 10^4 \)). The density starts from a definite value \( \rho_i = 0.518 \), which depends on the preparation process of the initial configuration, and grows to the final value \( \rho_f(\tau) \) reported in Fig. 2. Dashed lines are fits described in Eq. (3), and bold continuous lines are stretched exponential fits according to Eq. (5), as explained in the text. Their parameters are respectively reported in Figs. 4 and 6.

FIG. 4. Parameters \( \tau_0 \) and \( t_0 \) of fits from Eq. (3) for the density relaxation after a single "tap" depicted in Fig. 3, as a function of the logarithm of the vibration duration \( \tau \). Their (power law) behavior abruptly changes at \( \tau^* \), the characteristic vibration duration below which the final \( \rho_f \) is not affected. The two superimposed curves are fits described in Eq. (4).

FIG. 5. Bulk density \( \rho(t, \tau) \) relaxation, in a single vibration process, given in Fig. 3, rescaled according to Eq. (3), which is the continuous line in this picture, as a function of rescaled time \( (t - t_0)/\tau_0 \).

FIG. 6. Parameters of the stretched exponential fit, \( \tau_0 \) and \( t_0 \) [see Eq. (5)], from the fit of density relaxation in the long time region of a single vibration process, as a function of the logarithm of the vibration duration \( \tau \). Their behavior is analogous to the one described in Fig. 4.
FIG. 7. Static bulk density $\rho(t_n)$ from our MC data as a function of tap number $t_n$, for tap vibrations of amplitude $x_0 = 1.0 \times 10^{-4}$, $5.0 \times 10^{-3}$, $1.0 \times 10^{-3}$, $2.0 \times 10^{-3}$, $5.0 \times 10^{-3}$, $7.0 \times 10^{-3}$, $1.0 \times 10^{-2}$, $2.0 \times 10^{-2}$, $5.0 \times 10^{-2}$, $0.1$, $0.5$ (from bottom to top), and duration $\tau = 3.28 \times 10^1$. The continuous curves are logarithmic fits from Eq. (7), the parameters of which are given in Fig. 9.

process here explored corresponds experimentally to the density relaxation after a single application of vibrations to the box (a single “tap”).

B. Sequences of taps

In connection to recent experiments on compaction dynamics in granular media, we also studied the phenomena of density relaxation during a sequence of taps. Experimentally a “tap” is the shaking of a container filled with grains by vibrations of given duration and amplitude. In the following MC simulations, each single tap is a process in which vibrations of given duration and amplitude are applied to our particle on the lattice, according to a law $x(t) = x_0 = \text{const}$ for $t \in [0, \tau]$ and then the system is left to find a stationary state for a time $t_{\text{response}}$ in which $x(t) = 0$. So here $\tau$ is the duration of the vibration. After each tap we measure the static bulk density of the system $\rho(t_n)$ ($t_n$ is the nth tap number). We repeat the tapping sequence for different values of the tap amplitude $x_0$ and fixed duration $\tau$ (see Fig. 7). For this Monte Carlo experiment, which was very CPU-time consuming, we considered a system of size $30 \times 60$, averaged over $32$ different $\epsilon_{ij}$ configurations, and fix $\tau = 36.69$.

To describe experimental observations about grain density relaxation under a sequence of taps a logarithmic law was proposed in Ref. [15]:

$$\rho(t_n) = \rho_\infty - \Delta \rho_\infty / [ 1 + B \ln(t_n / \tau_0 + 1) ] .$$  \hspace{1cm} (6)

This law has proved to be satisfied very well by relaxation data in the present model [9], which can be excellently rescaled with experimental data using this four parameter fit as shown in Fig. 8.

In addition our MC data allow us to give more insight into the parameters of Eq. (6). After a few manipulations, Eq. (6) may be transformed into

$$\rho(t_n) = \rho_\infty - \ln(c)(\rho_\infty - \rho_0) / \ln(t_n / \tau_0 + c) ,$$  \hspace{1cm} (7)

where $\ln(c) = 1 / B$, $\tau_0 = \tau_1 / c$, and we have written in explicit form the asymptotic variation in density as the difference of a final asymptotic value ($\rho_\infty$) and an initial value ($\rho_0$), i.e., $\Delta \rho_\infty = \rho_\infty - \rho_0$.

FIG. 8. Experimental data from Knight et al. (square) and our MC data (circle) on density relaxation $\rho(t_n)$ in a sequence of taps, as a function of tapping number $t_n$, rescaled according to the logarithmic function given in Eq. (6).

We find that for a sequence of our MC taps of fixed duration $\tau$ and amplitude $x_0$, Eq. (7) is an excellent fit even fixing the parameter $c = 1.3$ as independent of $x_0$ (we now work at fixed $\tau$) and imposing that the fit function passes at $t_n = 0$ in $\rho_0 = 0.7388$, i.e., the measured static initial state density of our system, obtained from the prepared random starting configuration letting the particles just go down and their spin flip according the given rules. In this way we can reduce to just two the parameters to optimize in the fits with Eq. (6). The phenomenological parameter $c$ may be interpreted as the ratio of two typical times in the system ($\tau = \tau_1 / \tau_0$), and the finding that it is constant with the vibration amplitude $x_0$ in the range explored $x_0 \in [1.0 \times 10^{-4}, 0.5]$ suggests that in our model these are proportional to each other.

Our results for the density relaxation are shown in Fig. 7, and the values of the two fitting parameters $\rho_\infty$ and $\tau_0$ in Fig. 9. The parameter $\tau_0$, at fixed $\tau = 36.69$, seems to be a simple power law of the vibration amplitude $x_0$:}

FIG. 9. Fit parameters $\rho_\infty$ and $\tau_0$ for density relaxation from Eq. (7), as a function of vibration amplitude $x_0$. $\tau_0(x_0)$ seems to be well described by two simple power laws.
previous analysis, based on four experimental time series from

c  shows that it is possible to produce a good quality fit for these data

c onsidered as a constant. Our previous findings about the universality of

the vibration amplitude

c 0 776, K = 70, and X = 4 × 10⁻⁴. To show the qual-

ty of the fit, our Monte Carlo data corresponding to 11 dif-

c ent time series in a range of four orders of magnitude in

the vibration amplitude x₀ (x₀ ∈ [1.0 × 10⁻³, 0.5]), rescaled

g o according to Eq. (7) with these given two parameters values,

are presented in Fig. 10.

As stated above and proved in [15], Eq. (7) gives a good

four parameter fit for experimental data about density relax-

ation in a tapping sequence. Interestingly we find, moreover,

that it is possible to produce a good quality fit for these data also

with the parameter c considered as a constant. Our pre-

vious analysis, based on four experimental time series from

Knight et al. [15] (their lower capacitor measurements),

shows that c is bounded in a short interval. The average over

the previous values of c of the four series gives c = 1.4, and

with this fixed parameter we produced a good fit of data from

Ref. [15]. The fit is shown in Fig. 11 for these experimental
data, and its three parameters depicted in Fig. 12. For the two

intermediate vibration amplitude series it is also possible to

produce excellent fits with just two parameters, fixing ρ₀ to

the actual experimentally measured initial configuration den-

sity. A fit of experimental data with c = 1.3 is just slightly

poorer. It would be interesting to analyze other experimental
data after the insight of these theoretical results, and to pro-
duce measurements to better understand the limits of the

presented model and test its main forecasts.

It is a well known fact, in granular media, that the prop-

erties of the system in a given configuration strongly depend

on its preparation and past history. Specifically, if you make

a transformation, closing a loop in the parameters space of
the system, the final state strongly depends on the details of
the path and not just on the final values of the parameters.

Previous data show that this occurs in this lattice model too.

We performed also another Monte Carlo experiment with

our frustrated lattice gas model, which concerns short se-
quencies of taps in which the vibration amplitude x₀ is varied

at fixed amplitude increment Δx₀ holding constant their du-

ration τ. The process actually consists in a sequence of

2m−1 taps, of amplitude x₁, ..., xₙ ..., x₂m−₂, from an ini-
tial amplitude x₀ = x₂m−₁ to a maximal amplitude

xₙ = xₙ + (n−1)Δx₀ if n ≤ m and xₙ = xₙ − (n−m)Δx₀ if

n > m (see Fig. 13). This analysis allows us, moreover, to test

the validity of our previous findings about the universality of

the relaxation processes described by Eq. (7).

Our data for the system described above of size 30×60,
averaged over 32 different εᵢⱼ configurations, are depicted

in Fig. 14 for three different values of τ (τ = 3.67×10⁰,

3.67×10¹, and 3.67×10²) and four different values of

Δx₀ (Δx₀ = 0.025, 0.05, 0.1, and 0.2). We find that these
data may be described by a law formally equal to Eq. (7),

where the constant c is set again to c = 1.3, the same value

used before in Eq. (7). Here, in Eq. (10), the equivalent time

variable x is chosen to be x = xₙ for the increasing ramp, and

It is a well known fact, in granular media, that the prop-

ties of the system in a given configuration strongly depend

on its preparation and past history. Specifically, if you make

a transformation, closing a loop in the parameters space of

the system, the final state strongly depends on the details of

the path and not just on the final values of the parameters.

Previous data show that this occurs in this lattice model too.
to be \( x = x_f - x_n \) for the decreasing ramp, respectively. The starting density value \( \rho_0 \) is set, as in Eq. (7), equal to \( \rho_0 = 0.7388 = \rho(x_f) \) to fit the increasing amplitude ramp of the sequences (lower wings of Fig. 14), and to \( \rho(x_f) \) for the decreasing amplitude ramp (upper wings of Fig. 14).

The fits of our data with Eq. (10) are the superimposed continuous curves in Fig. 14, and the fitting parameters \( \rho_0(\tau, \Delta x_0) \) and \( \chi_0(\tau, \Delta x_0) \) are reported in Fig. 15. Our Monte Carlo data rescaled according Eq. (10) are reported in Fig. 16.

The characteristics of the present tapping sequence are very different from the others presented before, and so the good quality of this last scaling confirms the stability of the structure for the relaxation represented by Eq. (7). Moreover, the finding that in our model the constant \( c \) seems very robust to changes in the tapping process is surprising.

**IV. DIFFUSIVITY PROPERTIES**

To characterize the state of the packing and its capability for internal rearrangement, we studied particle self-diffusivity at fixed global density by setting \( x = 1 \). Specifi-
appear in this model. This corresponds to the known fact that
time correlation functions of Hamiltonian (2) have two char-
acteristic times at high densities (low temperatures) [14]. The
shorter is linked to the motion of particles inside cages of
other particles, while the longer corresponds to macroscopic
diffusionlike motion.

V. SPIN GLASS TRANSITION

The density $\rho_m$ interestingly coincides with the density at
which the spin glass (SG) transition of Hamiltonian (2) (for
$J \to \infty$) is located. This would imply that at $\rho_m$ the SG cor-
relation length $\xi_{SG}$ diverges, signaling the presence of col-
lective behavior in the system. In SG this length is infinite in
the whole region below the transition when it exists. Only
some quantities, as $\xi_{SG}$ or the nonlinear susceptibility $\chi_{SG}=(1/N)\sum_{ij} g_{ij}^2$, with $g_{ij} = \langle S_i S_j \rangle$, present clear
divergences at the SG transition. On the contrary, quantities
like the specific heat, the linear susceptibility, or the com-
pressibility have no divergent critical behavior and look
quite smooth around the critical point. In two dimensions
(2D), the SG and the diffusivity transitions occur at the high-
est possible density in the system. In that sense the transition
in 2D is only “dynamical;” one sees an effective transition
the position of which slowly shifts to larger densities for
longer observation times. We expect the same to occur in 2D
for the Reynolds transition too. The coincidence of the SG
transition and the suppression of self-diffusivity suggest a
 correspondence between the Reynolds transition in granular
media, the SG transition in magnetic systems, and the
“ideal” glass transition in glass-forming systems [7,14].

VI. CONCLUSIONS

We have studied a frustrated lattice gas linked to a spin
glass and to frustrated percolation, which gives agreement
with the dynamic behavior of granular media. It shows comp-
action and a typical logarithmic relaxation of density under
tapping, as found in experiments [15]. It moreover naturally
signals a correspondence between the Reynolds transition in
granular media and the spin glass transition in magnetic al-
loys [7,9]. The same model has also been exploited to give
microscopic insight into static stress distributions in disor-
dered granular systems, where agreement was also found
with experimental results [9]. Previously, without the gravi-
tational term in its Hamiltonian, it has been related to the
physics of the glass transition in glass forming liquids [14].

A common aspect, from the microscopic point of view,
which appears in all these seemingly different materials is
the existence of mechanisms leading to frustrations, as
quenched disorder of the spin interactions in spin glasses,
steric constraints, and the subsequent grain interlocking in
granular media or the formation of local arrangements of
molecules which kinetically prevent all the molecules from
reaching the crystalline state in glass-forming liquids. From
the thermodynamic point of view, the model studied here
clarifies the analogy between the role that vibrations play in
non-thermal systems as granular media and role of tempera-
ture in thermal systems as spin glasses or glass-forming li-
quids.

ACKNOWLEDGMENT

We thank IDRIS for computer time on Cray-T3D, and
M.N. the “Università di Napoli Federico II” for financial
support.

M. Jaeger, S. R. Nagel, and R. P. Behringer, Phys. Today 49,

Hansen (North-Holland, Amsterdam, 1993).

Mehta (Springer-Verlag, New York, 1994).

Majumdar, O. Narayan, and T. A. Witten, Science 269, 513