The Fractal Dimension of Stress Corrosion Cracks

V. K. HORVÁTH* and H. J. HERRMANN

HLRZ, KFA Jülich, Postfach 1913, 5170 Jülich, Germany

Abstract—We analyze pictures of experimental stress corrosion cracks in metals after digitalization and filtering and find that although they come from very different sources their fractal dimension is rather well defined to be \( d_f = 1.38 \pm 0.05 \).

In the rich morphology of crack shapes essentially everything can be observed. Wiggly non-fractal networks are seen on the surface of drying mud while glass can produce amazingly straight sharp lines. In many cases fractal shapes have been reported of varying fractal dimension [1–7] in particular by analyzing surfaces of various cracked metals and stones.

In parallel, numerical simulations have been performed (for an overview see [8]). Essentially elastic lattices with disorder were ruptured under a external loads (brittle fracture). Again different results for the fractal dimensions were reported depending on the type of load [9, 10], the elastic modes [11], the local breaking rule [12] and the connectivity of the crack [13]. Unfortunately, however, present numerical capabilities do not allow to calculate sufficiently large cracks to justify error bars of less than 0.1 in the fractal dimensions.

Since evidently different cracking processes produce different fractal dimensions it seems useful to concentrate of one experimentally well explored mechanism and to try to reconcile experimental and numerical data specifically for this mechanism. Stress corrosion cracking in metals [14–16] is a good candidate since it is known to be brittle to a very high degree. Also a lattice model has been proposed [12] for stress corrosion cracks in which counters on the crack surface take into account the damage inflicted by corrosion. This model generates fractal cracks also without noise and in this case rather precise estimates can be obtained for the fractal dimension \( d_f \) in two dimensions. In fact one finds that \( d_f \) depends continuously on a parameter \( \eta \) which describes the mesoscopic\(^{\dagger}\) breaking rule: the susceptibility for a bond to break increases like the local stress to the power \( \eta \). No \textit{a priori} derivation is known for the mesoscopic rules from the microscopic breaking mechanisms. Therefore \( \eta \) can only be determined as a parameter by comparing to experimental data. This is the aim of the present investigation.

Stress corrosion cracking is in fact a very treacherous phenomenon responsible for many plane accidents and because for its technological importance has been studied quite extensively particularly by the airplane industry in the 70ties. In stress corrosion cracking a chemical agent impregnates the microfractures and corrodes the crack tips where the stress

*Present address: Institute for Technical Physics P.O. Box 76, Budapest, 1325 Hungary
\( \dagger \)mesoscopic = level on which elastic equations are valid.
intensity is highest. So the crack grows at a lower applied stress with a velocity controlled by the chemical reaction. Usually ductile materials become brittle. Two-dimensional cuts through stress corrosion cracks in alloys as shown in Figs 1 and 2 are typically tree-like and quite branched. These tree or river like patterns are typically observed during low energy cracks which propagate along well defined low-index crystallographic planes known as cleavage planes. Because of this the network of the lines formed by the $d-1$ dimensional cross-section of the crack is connected and the fracture patterns can be considered as embedded into two dimensions. (It is known that a $d_f$ dimensional fractal embedded into a $d$ dimensional space usually leads to $d_f(d-m)$ dimensional object, where $m$ denotes the dimension of the projection slice.) Although they seem to be good candidates for being fractal, a systematic analysis of such cuts has not yet been performed to our knowledge.

Fractal image analysis is based on a covering method [18] where using tiles of a particular geometrical shape, one has to measure the area defined by the number of tiles

![Fig. 1. Intergranular stress corrosion cracks taken from [17]; (a) for Inconel 600 in high temperature water; (b) for Monel 400 in steam, (c) is result obtained from (b) after filtering.](image1)

![Fig. 2. Stress corrosion cracks showing macroscopic branching taken from [14]; (a) forging of high strength aluminum alloy 7079-T6, which was locally recrystallized; (b) and (c) ternary Al–Mg–Zn alloy exposed to aqueous halide solutions (b) at high stress intensity, (c) at low stress intensity. These pictures have already been filtered.](image2)
covering the object, for tiles of different linear sizes. Different ways to perform this covering are the Minkowski–Bouligand Method (MBM) [19], the Horizontal Structuring Element Method (HSEM) and the Variation Method (VM) [20]. These algorithms have been tested and compared [20]. It turns out that the VM algorithm gives the most precise value of the fractal dimension \( d_f \) for fractal functions but it is useless in the case of polyvalent functions like crack images. The comparison [21] of the Box Counting Method (BCM) to the Sand Box (SB) method indicates that the SB method gives better estimates of the generalized dimension than the BCM, due to the necessary averaging this method requires considerably more computing time than a naive application of the BCM. In this paper we will use the BCM with some modifications.

In the original BCM one covers the object with boxes of same size. The simplest way to that is to place a grid on the photo dividing it into boxes. If parts of the crack lie inside a box it is called ‘filled’ or ‘1’ otherwise ‘empty’ or ‘0’. The number of boxes covering a unit area depends on the size \( R \) of each box to the grid. The dependence of the number \( N \) of filled boxes on \( R \) is: \( N \propto \frac{1}{R^{d_f}} \).

To calculate the fractal dimension of real stress corrosion on a computer we first digitized their photos by an image processor card connected to a high resolution video camera. Since the resolution of this system depends on the camera–object distance, we moved the objective of the camera as close to the pictures as possible before observing optical distortions in the digitized image. These distortions can be seen through the appearance of kinks in the image of straight lines. In the case of our camera it defined a resolution of about 700 dots per inch (dpi). To be on the safe side we calibrated our digitizer system to a maximum of 650 dpi and we used this resolution as the unit in which we measured the grid size. Standard filter algorithms were used to supress the noise from the digitized pictures [22]. In Fig. 1 we see an example for the effect of this filtering.

Each box in the unit-grid contains either 0 or 1. First we covered our object using a grid of \( R = 1 \). Counting the number of matrix elements of value 1 we obtain \( N(1) \). Next we construct larger cells sticking together unit cells of sizes \( 2^2, 3^2, \ldots, R^2 \) and obtain analogously \( N(2), N(3), \ldots, N(R) \). In the case of scale invariant objects, if \( \ln(N) \) is displayed versus \( \ln(l) \) where \( l = R/R_{\text{max}} \), the resulting graph will be a straight line and its slope gives the fractal dimension.

The construction of a larger grid from the unit-grid is only defined modulo translations. Typically we may construct \( R^2 \) different grids with same size \( R \), labeled by \( i \), by shifting the grid from the top-left hand side corner by up to \( R \) unit-grids in any of the two directions. The best way to do this is to count \( N'(R) \) for all these shifted grids \( i \) and take the average \( \overline{N}(R) = \langle N'(R) \rangle_{i=1\ldots R^2} \).

Although this method is very comfortable to use, it involves some artificial effects:

1. By averaging over these shifted positions one can get better statistics, but because the number of possible different positions increases with the grid size the quality of the statistics also depends on \( R \). In the case \( R = 1 \) we have only one possible grid position while for grid size \( R \) we have \( R^2 \) different positions.
2. The number of filled boxes can only take integer values, thereby inducing jumps when the grid size is changed from \( R \) to \( R + 1 \).
3. The entire size of the grid must be a multiple of \( R \) otherwise one either looses data points or one gets spurious jumps in the log–log plot specially when \( R \) is large [19].

To take into account the effects 1–3 we combined the simplicity of the BCM with ideas of the Variation Method by moving the camera (from the object) to the position where the resolution is 2, 3, \ldots, \( R \) times worse as compared to the optimal case instead of constructing larger boxes based on the unit-grid. For all these camera–object distances we
have taken the same number of digitized images but in different positions, randomly shifting and rotating the object picture. We want to point out that since we are analyzing experimental images and not computer generated pictures we can rotate our photos under the camera by any angle.

Since the lines of the cracks have a certain width, the definition of \( N(R) \) is not obvious for small \( R \) when the wide lines themselves are covered by several boxes. To handle this problem we may divide for a given grid of size \( R \) the boxes into different classes.

1. All the filled boxes having empty neighbours are defined as the border of the object and labeled as ‘class \( B(0) \).’
2. All the filled boxes having only filled neighbours and at least one \( B(0) \) class neighbour are then in class \( B(1) \).
3. Following the construction we define a filled box to be in class \( B(q) \) if it has at least one neighbour in class \( B(q-1) \) and does not have any neighbour of a class \( B(k) \) with \( k < q - 1 \).
4. Just for the full definition of the hierarchy we may define the boxes of class \( B(-1), B(-2), \ldots B(-q) \) using ‘empty’ instead of ‘filled’ in points 2 to 3.

This hierarchy of boxes covers the entire space and can characterize objects more sensitively [23] than the simple covering in the BCM. One can recover the number of boxes covering the object in the BCM as

\[
N = \sum_{i=0}^{\infty} N(B(i)).
\]

We have calculated \( N^i(R) \) and displayed \( \ln(N) \) \( v \) \( \ln(l) \) with \( l = 1/R \) as shown in Fig. 3 for the two cracks of Fig. 1. Various shifts and rotations of the photos under the camera give slightly different curves, giving a good measure for systematic deviations due to the method. Over at least two orders of magnitude* one sees straight lines and their slopes are \( d_f = 1.40 \) and 1.36 for Figs 3(a) and (b) respectively. Analogous plots for the three cracks in Figs 2(a)–(c) give 1.32, 1.40 and 1.40. The error bars are about 5%. All these values are very close to each other suggesting that all cracks have the same fractal dimension which would then be \( d_f = 1.38 \pm 0.05 \).

![Graphs](https://via.placeholder.com/150)

Fig. 3. Log–log plot of \( N \) against \( l = R/R_{\text{max}} \) for the two cracks of Fig. 1 in the same order. The circles correspond to \( N(B(0)) \) and the stars to \( N = \sum_{i=0}^{\infty} N(B(i)) \).

*Note that the axis of Fig. 3 are given in natural logarithms.
Fig. 4. Different printer outputs were used to test our algorithm. The deterministic stress corrosion crack (a) with parameter $\eta = 0.2$ was taken from [12]. Koch curve (b) with $r = 1/2.1$ sidewall has a fractal dimension $d_f = (\ln(4)/\ln(2.1)) \approx 1.868$. In both cases the discrepancy from the known value is less than $\pm 3\%$.

To test our algorithm we have also considered a deterministic crack taken from [12] and a Koch pattern with known fractal dimension. We have processed the printer outputs (Fig. 4) in the same way as the experimental photos. In both cases the discrepancy from the known value is less than $\pm 3\%$.

Concluding we analyzed various experimental stress corrosion cracks in alloys and observed that they are all fractal and have roughly the same fractal dimension of $d_f = 1.38 \pm 0.05$. Comparing these values to the theoretical predictions of $d_f$ as a function of $\eta$ from [12] one sees that $\eta$ would lie between 1 and 1.2 if the damage caused by the corroding agent has a finite life-time. If the damage is permanent $\eta$ would be about 20\% lower. It would be interesting to analyze more pictures of stress corrosion cracks obtained under a large variety of conditions to test further our claim for the universality of the fractal dimension. From the theoretical side it would be interesting to understand why the local breaking rule has an $\eta$ of the order of unity, which in fact the exponent one derives from first principles (Gibbs–Thompson relation) for the problem of solidification from an undercooled melt which is a scalar analog to the moving boundary problem that can be formulated for the growth of cracks in an elastic medium [24].

REFERENCES


