1. Introduction to basic notions and facts

Hans J. Herrmann\textsuperscript{†}

This first chapter is particularly simple. The main goal is to give physicists an idea about the phenomenon of fracture without entering into many details. I hope the experts will allow for the many oversimplifications, omissions and the lack of precision. The reader who wants to know more can read one of the many textbooks\cite{1} that have been written on the subject or consult the classical collection of review articles edited by Liebowitz.\cite{2}

1.1 Objectives

Every solid breaks if a sufficiently large load is applied on it. The value of this load as well as the shape and other characteristics of the resulting crack strongly depend on the material and on the way how the load has been applied. To illustrate how different situations can arise we show in figure 1.1 four different shapes of cracks obtained under various conditions and for different materials. Fig. 1.1a shows the crack of an alloy that started on a free surface obtained under an uniaxial tension in a corroding environment. Fig. 1.1b shows various stages of the breaking of a thin sheet of clay under shear. In fig. 1.1c a drop of acid was placed on the surface of a plexiglass plate some time ago and fig. 1.1d shows the surface of a drying block of concrete.

In order to understand why and how a given sample breaks one can pursue various approaches depending essentially on the length scale in which one is interested. As depicted in figure 1.2 several disciplines are involved in this enterprise. Ranging from atomic scales to the size of grains the mechanisms involved in fracture are very diverse and strongly vary from one material to the other. Generically these microscopic scales are studied by material scientists, in fact, there are at least three different families of materials: metals, rocks and polymers each studied by its own community. On human scales, i.e. from one centimeter upwards fracture is a problem of civil and mechanical engineering. In this range the aim is the design of structures such as to prevent their failure and this is achieved by relying heavily on experience. In the intermediate range, i.e. between microns and centimeters

\textsuperscript{†} Service de Physique Théorique, CEN Saclay, F-91191 Gif sur Yvette Cédex, France.
Figure 1.1 (a) Intergranular stress corrosion crack of Inconel 600 in high temperature water, taken from ref. [3]. (b) Four stages of the development of a fault network on the surface of a sample of wet clay under shear, by Z. Reches from ref. [4]. (c) Surface of a plexiglass plate after the reaction of a drop of acid by G. Ananthakrishna, Kalpakam, India. (d) Surface of a 5 cm thick plate of drying concrete by P. Acker, (LCPC) and L. Oger, (ESPCI), Paris.

Figure 1.2 Schematic representation of the study of fracture on different lengths scales.

the mechanical behaviour can be described rather well by continuum equations in which the material is just characterized by some parameters and where only few types of behaviour, like "elastic", "plastic" or "viscoelastic", are distinguished. In this book we will mainly focus on this intermediate regime and most of the concepts that will be introduced will apply to this regime.

The fracture of an ideal crystal can be handled theoretically and various properties can be calculated from first principles. Unfortunately, however, the force needed to break the crystal that one obtains from these calculations is already
several orders of magnitude larger than forces measured experimentally on real materials. The reason for this discrepancy is that real substances have “disorder”, i.e. they have deviations from the perfect crystal structure and it happens that the process of fracture is extremely sensitive to disorder. Why this is so and which are the macroscopic effects of disorder on fracture is one of the central issues of this book.

Microscopically disorder can mean many things: vacancies, inclusions, composite structure, dislocations or even grain boundaries but on the length scales we are interested in, these spatial inhomogeneities can be reduced to a random noise in the material properties or, on the computer, to sequences of (quasi-)random numbers. Cast into this form randomness is familiar to statistical physicists. It is known that in such a formalism one can find critical phenomena, like “percolation”, one can see “fractals”, specially if one has an unstable growth phenomenon and one can encounter unusual statistics, like “non-self-averaging” quantities and “multifractal” probability distributions. How concretely all this appears in fracture that is the content of this volume. Chapter 4 is devoted to introduce the notions of disorder.

1.2 The mechanical response of a solid

Before discussing the breaking of a solid it is useful to understand the mechanical behaviour of a flawless, homogeneous medium. On length scales on which one does not feel any more the chemistry or the texture of the material its mechanical behaviour is determined by a few quantities like the “toughness”, the “yield point” or the “elastic moduli” which enter in what are called the “constitutive relations” or “rheological laws”. These relations tell how the material reacts to a local application of a force. A well-known example for such a relation is Hooke’s law. Constitutive relations are directly obtained from the experiment.

Let us consider the classical experiment by Young. A homogeneous bar of length $L$ and cross section $w \times w$ is pulled in the direction of the length with a force $F$, i.e. submitted to a uniaxial tension. Due to the load the block is elongated by $\Delta L$ and its width decreased by $\Delta w$ (see also figure 1.2 of chapter 3). The relative elongation $\delta = \Delta L/L$ can be measured as a function of $F$ and in fig. 1.3 we see three typical situations.

For small elongations the relation between $F$ and $\delta$ is usually linear and reversible and one can apply the formalism of linear elasticity which will be discussed in chapter 3. An isotropic medium is characterized in this linear theory by just two elastic moduli, the “Young modulus” $E$ and the “Poisson ratio” $\nu$ defined by Hooke’s law $\sigma = F/w^2 = E\delta = -(E/\nu)(\Delta w/w)$ where $E \geq 0$ and $-1 \leq \nu \leq \frac{1}{2}$ for thermodynamical reasons; $\sigma$ is called the “stress”. For simplicity most of the fracture models in this book will be considered in this linear approximation only.

If the force is increased beyond a certain material-dependent value $F_N$ one sees deviations from the linear law. In some specific cases it can even happen that
the relation is non-linear already for infinitesimal forces due to force-dependent internal contacts ($F \propto \delta^{3/2}$ is Hertz's law for spheres and $F \propto \delta^{3-4}$ is found for a random packing of cylinders). In these cases the elastic moduli are generalized to be force-dependent functions and one calls $k = dF/d\Delta L$ the "stiffness" and its inverse the "compliance" of the system. Numerically these cases can be handled by inserting the experimental, non-linear constitutive relation into the relaxation algorithms that one also uses to solve the linear equations.

Up to now we only encountered "elastic" or reversible response, i.e. that the system goes back to exactly the original shape when the force is reset to zero. If a certain material-dependent force $F_Y$, called the "yield point", is passed this reversibility is lost and one finds "plastic" behaviour. Beyond $F_Y$, let's say at A in fig. 1.3a, a finite, permanent elongation, called "plastic deformation" or "dilatancy", remains when the force is reset to zero which is the point B in fig. 1.3a.

Plasticity is due to flow inside the material, commonly flow along crystal planes. In the ideal case of "perfect plasticity", i.e. when $dF/d\Delta L = 0$ beyond $F_Y$ as shown in fig. 1.3c, the flow is indistinguishable from that of some fluids. In fact there are "non-Newtonian" fluids like "Bingham fluids", which need a threshold pressure (more precisely shear stress) to flow, analogous to the yield point so that for these particular cases the distinction between fluid and solid becomes rather semantic.

In the majority of cases, however, one has a finite "strain-hardening", i.e. $dF/d\Delta L$ is non-zero beyond $F_Y$ as is the case in figs. 1.3a and 1.3b. Strain-hardening implies that once $F_Y$ is passed and the force is reset to zero and increased again the system becomes stiffer as can be seen from fig. 1.3a: The curve BA is steeper than OA. The internal flow has modified the material and made it harder, a process of technological importance. In some cases as the one shown in fig. 1.3c an extra force is needed to unleash plastic flow, in fact inside the material, bands of dislocations ("Lüders bands") are set in motion at this point. The resulting instability is, however, rapidly controlled by the strain hardening.
The mathematical formalism needed to describe plasticity has to include the dependence on the history of force applications and the fact that for any finite dilatancy the system is elastic as long as the applied force does not exceed $F_Y$. This non-linear formalism which in its present form was only developed in the last decades will be described with more detail in chapter 3 but since it is rather heavy it has not yet been implemented into the statistical fracture models that this book is concerned about. Plasticity is, however, a very common phenomenon and the generalization of the approaches presented in this book to the plastic case is an important task for the future.

The deformation of a solid can cause energy dissipation, e.g. heating, and as a consequence the response of the system will not be immediate but delayed (see also figure 1.3 of chapter 3). This effect, which is called “viscoelasticity” or “viscoplasticity” depending on if one is below or above the yield point, introduces at least one characteristic delay time $\tau$ into the problem. Introducing $\tau$-dependent material constants one can take into account these effects so that for instance in Hooke’s law the force depends on elongations made over a time interval ($\sigma(t) = \int_0^\infty C(\tau) \delta(t - \tau) d\tau$ where $C(\tau)$ is a material-dependent delay kernel). Applying periodic forces (or imposing periodic elongations) with a frequency $\omega$ allows to measure the dissipation. In the linear case the frequency-dependent formalism is completely analogous to the case of highly viscous fluids and in fact for $\omega \tau \ll 1$ these fluids behave like elastic solids (see e.g. the clay discussed in chapter 2). Again we are on the borderline between fluid and solid.

1.3 Phenomenology of fracture

1.3.1 Brittle and ductile

When one reaches a certain force $F_c$ while performing Young’s experiment the sample will break apart. If this happens before getting to the yield point $F_c \leq F_Y$ the system behaves elastic till the breaking and the fracture is called “brittle”. In the opposite case $F_c > F_Y$ the sample breaks in the plastic regime and the fracture is called “ductile”. Typically a fresh potatoe chip breaks brittle and a chewing gum ductile (at room temperature). Brittle or ductile fracture are not just properties of the material but depend also on temperature or pressure. In 1911 von Kármán discovered that Carrara marble which is usually brittle becomes ductile when submitted to hydrostatic pressure (see fig. 1.4b). The force at which that happens is about three times larger under tension than under compression. Brittle to ductile transitions were equally found increasing the temperature or decreasing the speed of the deformation, or through the presence of corroding agents.

It is again useful to consider the dependence between force $F$ and elongation $\delta$ which in the case of fracture is also called the “breaking characteristics”. In fig. 1.4a we see the transition from brittle to ductile as obtained from the experiment on Carrara marble by showing the characteristics at different hydrostatic pressures $\sigma_1$. For $\sigma_1 = 0$ and $\sigma_1 = 10 \text{ MPa}$ one has brittle fracture and the charac-
Figure 1.4 (a) Breaking characteristics for marble under compression for different hydrostatic pressures $\sigma_1$. The uniaxial excess force $F$ is defined in (b) where $A$ is the top (or bottom) surface of the sample on which the force is applied.

teristics terminates rather abruptly at rather small elongations. Therefore technologically speaking brittle fracture is dangerous. At high pressures of $\sigma_1 = 100$ MPa fracture is ductile and it occurs after the sample has been substantially elongated. 1 MPa = 1 megapascal = $10^6$ N/m$^2$ is in this field the common MKSA unit used for elastic moduli, pressure and strength of materials. Table 1.1 shows how it is related to other units that appear in the literature.

In both cases, brittle and ductile, the characteristics follows for small forces the constitutive relation of the material. For larger forces the material looses stiffness due to changes that occur inside the material before fracture. At a force $F_c$ the curve reaches its maximum and after that the sample elongates and breaks without any increase in the externally applied force $F$. The regime after the maximum, called “postfailure” regime or also sometimes called “catastrophic” regime in this book, can be obtained experimentally by imposing the elongation of the sample instead of imposing the force. $F_c$ is the technologically important “breaking force”.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kbar</td>
<td>100 MPa</td>
</tr>
<tr>
<td>1 dyn/cm$^2$</td>
<td>0.1 Pa</td>
</tr>
<tr>
<td>1 kp/mm$^2$</td>
<td>981 MPa</td>
</tr>
<tr>
<td>1 psi</td>
<td>6894.5 Pa</td>
</tr>
</tbody>
</table>

Table 1.1 Units of pressure (psi = pounds per square inch).

In the case of brittle fracture the characteristics in the postfailure regime can show very strong variations from sample to sample and for a given sample the curve is seldom smooth. One distinguishes postfailure behaviour of class I and of
class II depending if the characteristic bends back or not\cite{10,11} as seen in fig. 1.5. Tennessee marble is typically of class I and basalt of class II. In order to measure the characteristics for class II one needs a very stiff testing machine and has to continuously and very quickly adapt force and elongation via a servo-control.\cite{12}

Brittle fracture is the main subject of this book. It is therefore our aim to understand why the characteristics has the given shape and how it depends on the sample size, the disorder or the mechanical material constants. Experimental facts concerning the brittle breaking characteristics are scattered throughout the engineering literature. Let us just mention here that for uniaxial compression of bar-shaped rocks the postfailure stiffness roughly increases linearly with the diameter and with the length of the bar.\cite{10}

In ductile fracture very different phenomena appear, the most spectacular being geometrical instabilities. Let us consider again Young’s experiment where under uniaxial tension an instability called “necking” occurs: After passing the yield point flow occurs on certain crystal planes rendering the bar slightly thinner at some regions as compared to the rest (“localization”) as sketched in fig. 1.6a. Since in these regions the cross section is smaller the force per area is larger and more elongation, i.e. more flow of crystal planes will take place. Consequently the region will become thinner and thinner and the force per area will finally diverge so that the sample tears apart at this neck. This is what happens when we stretch a chewing gum. Another well-known instability, called “buckling”, occurs under uniaxial compression: The axial symmetry is broken and the bar bulges into one direction as seen in fig. 1.6b. Buckling is the main source of failure of cellular structures like bee-hives.\cite{13} The geometrical instabilities of plasticity are a subject of much current interest\cite{14} on one hand as a mathematical problem (“Hadamard instabilities”) and on the other hand because of the variety of phenomena they can generate. However they are beyond the scope of this book.
Figure 1.6 Geometrical instabilities in ductile fracture: (a) necking, (b) buckling.

Figure 1.7 The three modes of loading.

1.3.2 The three-dimensional nature

For the sake of simplicity we have only considered up to now uniaxial experimental setups. In real applications one has, however, seldom an axial symmetry of the load, usually both the externally applied forces and the resulting deformations of the solid are complicated three-dimensional vector fields. To take this into account the mechanical response of the solid must now be described by a tensorial formalism. The force is replaced by the “stress tensor” and the elongation by the “strain tensor”, chapter 3 will deal with this in much detail. Conceptually, however, not much is changed as compared to the uniaxial case.

The three-dimensional nature of the mechanical loads affects of course also fracture. On a crack three modes of loading can be distinguished as shown in fig. 1.7. Any arbitrary load can be expressed in a linear theory as a superposition of these three modes. Details about this can be found in any textbook on fracture.\cite{15}

The breaking force and the characteristics depend of course also on the type of external load. Besides uniaxial tension and compression we will encounter in this book cases of shear, uniform dilatation and radial tension with angular symmetry.
Since breaking is essentially due to the growth of a crack at its tip it is possible to reduce the breaking force \( F_c \) to a quantity that describes the stress that is needed at the tip to open the crack further. This material-dependent quantity \( K_c \), which is called "toughness", will be defined in the next section.

1.3.3 Dynamical effects

Important in real applications are also time-dependent loads. A very common phenomenon is "fatigue".\(^{[1]}\) A body is submitted to the same load many times and nothing particularly seems to happen; suddenly when the load is applied for the \( N \)th time it breaks. If the load is periodic in time, e.g. sinusoidal, one calls this phenomenon "cyclic fatigue" and the number \( N_f \) of cycles for failure to occur is also called the "failure life". The dependence of \( N_f \) on the amplitude \( S \) of the periodic load is schematically shown in fig. 1.8 for the case of metals. We see that if the amplitude is below the "endurance limit" \( S_e \) the sample will not break at all. An empirical relation, also called "Besquin relation", of the form \( N_f \propto (S - S_e)^{-a} \) has been found to hold for exponents \( a \approx 8-20 \). Except for frequencies above 1000 Hz, frequency has a negligible effect on fatigue in metals. The curve of fig. 1.8 and therefore also \( S_e \) depend not only on the material but also on the type of load. For uniaxial tension the endurance limit is in general much larger than for shear.

Shocks are another common case of time-dependent loads. They can for instance be generated by hitting the sample with a hammer and are also encountered in explosions. Usually at the shock the energy that is locally furnished exceeds many times the energy necessary for fracture. Partly this energy goes into the kinetic energy of the propagating cracks, which therefore propagate very fast. The excess energy produces crack-branching and fragmentation, both phenomena of high current interest which will come up again in this book.

Another important time-dependent effect of fracture is "stress corrosion
cracking". It was discovered\textsuperscript{[16]} that the toughness of glass is decreased by 20% if it is put into a humid environment. The reason is that water penetrates into the glass at the crack tip because there the crystalline structure is more open. Inside the glass the water forms a base with existing sodium ions and this basic liquid corrodes the crack tip region lowering its toughness and enhancing the possibility for brittle fracture. It turned out that stress corrosion cracking is in most cases the responsible mechanism for the tearing of the aluminium or titanium alloys that are used to build airplanes and is therefore an important source of accidents. For this reason industrial and military agencies have made an important effort to better understand the phenomenon.\textsuperscript{[17]} This is also the reason why it is forbidden to take mercury or thallium on a plane.

Many possible mechanisms leading to stress corrosion cracking have been found and they are either anodic or cathodic, i.e. the effect can be suppressed in an electrolytic environment by having either the anode or the cathode on the sample and the corroding agent as electrolytic medium. The most common cathodic process is hydrogen embrittlement of metals in which at the crack tip the metal is ionized due to charges that recombine with $\text{H}^+$ ions from the corroding medium at the outer surface of the sample. The strength of the ionized metal is lower and therefore it is more brittle. A typical anodic effect happens at metals that are protected from the corroding agent by a coat of oxyde which is ripped open at the crack tip exposing the pure metal to the agent just at the tip.\textsuperscript{[18]} In all cases the velocity of the crack is controlled by the chemical reaction and is therefore rarely greater than 0.1 m/s. This slow crack propagation can sometimes be observed on the windshield of the car and also the crack shown in fig. 1.1c took several days to be grown.

1.3.4 Statistical effects

For most materials the sample to sample variations of the brittle fracture strength are very strong and a statistical treatment is useful. The most commonly used form for the cumulative probability $p$ of fracture is the "Weibull distribution".\textsuperscript{[81]}

$$p = 1 - \exp\left(\frac{(\sigma - \sigma_u)^m}{\sigma_0}\right),$$

(1.1)

where $\sigma$ is the externally applied stress and $\sigma_u$ is the largest stress at which the probability of failure is zero and which is usually equal to or less than one-half the value of the mean critical stress. $m$, $\sigma_0$ and $\sigma_u$ are material constants but they vary substantially with the surface condition, sample preparation and temperature, up to the point that for a given alumina $m$ can take values between 3 and 12. In fig. 1.9 we see experimental data for the distribution of strength for high-density alumina specimens.
3. Phenomenology of fracture

Figure 1.9 Distribution of strength values for the alumina Lucalox (dots) fitted by a Weibull distribution of the form of eq. 1.1 (full line).
1.4 The process of fracture

It is rather evident that fracture occurs through the propagation of cracks but why do these cracks appear and how do they organize to break the system? In the following we will describe qualitatively some of the most common processes that yield to brittle fracture.

1.4.1 The behaviour of one microcavity

As already mentioned before real materials have plenty of heterogeneities which can range from very localized interstitial vacancies to rather extended grain boundaries. At length scales at which the medium can be approximated by an elastic continuum we can disregard the detailed nature of these heterogeneities and generically consider that there are randomly distributed "microcavities" in the medium. Usually one distinguishes two types of microcavities: "pores" which have roughly the same length in all directions and can be approximated by spheres or ellipsoids, and "microcracks" which are typically $10^2$ to $10^4$ times longer than wide, one says, they have an "aspect ratio of $10^2$ to $10^4$". In rocks, pores can be due to intergranular spaces in sediments or to fluid inclusions in igneous rocks while microcracks can be cleavages through mineral grains or lie inside grain boundaries.

The next useful step is to understand how a microcavity modifies the mechanical behaviour of a solid. So let us perform Young's experiment for a sample with one cavity (figs. 1.10b and c) and compare it to a sample with no cavity (fig. 1.10a). In the homogeneous sample of fig. 1.10a every volume element contributes equally to the elastic response or more precisely, on any cubic volume
element $\Delta V$ act three pairs of forces in opposite directions having for each volume element the same absolute value $F$. If $\Delta S$ is the surface of one face of $\Delta V$ then $\sigma = F/\Delta S$ is the stress so that one can say that in fig. 1.10a the stress is everywhere the same.

If one drills now a cavity into the medium those volume elements that would have been inside the cavity cannot contribute anymore to the elastic response and the remaining volume elements must take over the excess load, i.e. the stress that would have acted on the missing volume elements. It happens, however, and this is crucial for fracture, that the excess load is not shared equally by the remaining volume elements but that the redistribution of stress is strongly inhomogeneous.

It is already intuitively clear that since at the height of the cavity ($h$ in fig. 1.10) the cross section is reduced the stress acting on volume elements at this height $h$ will be enhanced. But the increase of stress is not the same for all volume elements on height $h$. If $\sigma_0$ is the stress acting on the volume elements before the cavity was drilled then the enhanced stress acting on volume element $k$ is $\sigma_k = K_T \sigma_0$ and we call $K_T$ the “stress concentration factor”.

For a cylindrical hole in an infinite, linearly elastic medium it can be shown analytically that next to the hole at the height $h$ (point A in fig. 1.10b) one has $\alpha_k = 3$ independent of the cross section while $\alpha_k \to 1$ for volume elements at height $h$ that are far from the hole. For an ellipsoid with the longer axis of length $2a$ perpendicular to the load one has

$$\alpha_k = 1 + 2\sqrt{a}/\rho$$

at point A in fig. 1.10b where $\rho$ is the radius of curvature at point A.

For two-dimensional plates analytical calculations can be pushed much further using conformal transformations in a complex formalism. So it is known that for an elliptical hole in an infinite, linearly elastic medium at height $h$ the excess stress, $\sigma_k - \sigma_0$, decays like $1/\sqrt{r}$ close to point A while far from the hole the decay is that of a dipole field, namely $1/r^2$ where $r$ is the distance from point A. More about this will be said in chapter 6. Close to A one can even calculate for each of the three modes of fig. 1.7 the full dependence on the angle $\theta$ as shown in fig. 1.10b for each of the components of the stress. These formulas can be found in the textbooks.\[^{[15]}\]

Close to the tip of a microcrack like the one shown in fig. 1.10c the excess load also decays in a plate like $1/\sqrt{r}$. The proportionality constant $K$, defined through $\sigma_k - \sigma_0 = K/\sqrt{2\pi rf(\theta)}$ is called the “stress intensity factor” and it depends on the length of the crack, the geometry of the sample, etc. The value of $K$ at which the crack breaks at the tip is the material-dependent constant, called “toughness” $K_c$ which was mentioned already before. For a given crack tip one has three toughness values, one for each of the three modes of fig. 1.7. Knowing the toughness of a sample is of more use to engineers than knowing just the breaking
force in a specific experimental setup because the breaking force depends for instance on the length of the crack, the geometry, etc. The typical toughness of steel in mode I is $K_{IC} = 5-10$ GPa $\sqrt{m}$.

We see from the enhancement factor of an elliptical hole at point A in eq. 1.2 that the sharper the tip of a microcrack the more its stress is enhanced at the tip. At an infinitely sharp tip the stress has in fact a singularity. So, even if the externally applied stress $\sigma_0$ is much below the breaking stress $\sigma_c$, locally, namely close to the tips of microcracks, the stress can be much higher than $\sigma_c$ and so the material can break at these tips. Once a microcrack tears open at its tip it becomes longer and therefore the stress at its tip is enhanced even more so that it will not stop growing anymore. We see that within this picture crack tips are unstable.

In reality a mathematical divergence of the local stress at a cusp-like tip does not exist. On one hand the continuum description is only valid up to a lower cutoff at which the atomistic nature becomes visible. Consequently one cannot get arbitrarily close to the crack tip neither can a tip become arbitrarily sharp. On the other hand, if the value of the local stress passes beyond the yield point it is certainly not allowed to consider anymore linear elasticity but in fact the material becomes plastic. One therefore has the interesting scenario that although the sample as a whole is elastic and breaks brittle there are regions inside the material, namely around the tips of microcracks, in which the material deforms plastically and breaks ductile. In fractography, the experimental technique that investigates the physionomy of the crack surface, one can nicely visualize the two types of behaviour occurring in the same sample.

The shape and the mechanical response of the plastic regions around crack tips surrounded by an elastic medium is difficult to estimate. However, only the characteristic size $\xi$ of these regions is essential for fracture because at $\xi$, which is the shortest distance from the tip of the crack to the boundary of the plastic zone, the singularity of the elastic stress field is cut off. The largest value $\sigma_c$ of the resulting stress field which is necessarily on this boundary has to be compared to the toughness of the material if one wants to know when the crack will grow. In reality there are even more effects that also introduce a cut off on the singularity of the stress field, like viscoelasticity or disorder and these effects must also be taken into account if one wants to know the critical stress. In numerical calculations such a cut off is naturally introduced through the discretization of space. We will discuss this point in more detail in chapter 5.

1.4.2 Interacting cracks

Up to now we have only discussed the behaviour of one single crack. In fact, looking at fracture in the real world one gets the impression that most samples are broken by just one crack. On the other hand, except when for test purposes a crack is artificially nucleated by a notch before applying the load, in most materials no particular crack can be singled out to be the critical crack that will break the system and on which the toughness etc. could be measured locally. In fact we
started out with a material full of pores or microcracks and therefore the next, and perhaps essential point to understand fracture is to follow how out of many essentially equivalent randomly distributed microcavities one crack will be born that will tear the system apart. This enterprise is difficult and has been neglected very much in the past but it is, at least for brittle fracture the way to understand how the crack responsible for failure comes into existence.

Imagine an ensemble of microcavities of arbitrary shape and orientation scattered randomly in space as shown in fig. 1.11. As discussed above each cavity produces a stress enhancement field around it which depends on its geometry and size and decays at large distances like a dipole field. Because of this slow decay (slower than exponential) technically one has a long range interaction.

As long as one only considers linear elasticity the stress enhancement fields coming from the various microcavities can be added together (superposition principle) and one is left with an extremely complex stress enhancement field for the ensemble which even numerically cannot always be handled. This problem will come up again in several chapters (6, 7, 9) of this book.

The interplay of several stress enhancement fields can yield to the most diverse effects: Between two close cracks that are aligned and perpendicular to the load like the pair shown in fig. 1.11 at A the stress field is particularly enhanced. For long, straight cracks one can calculate that at the tip of one single crack of length $a$ the enhancement is proportional to $\sqrt{a}$ while between two cracks of length $a$ the enhancement is proportional to $a$. Consequently the two cracks shown at A in fig. 1.11 will attract each other and if the external load is large enough coalesce, i.e. the neck between the two will break and they will become one single crack.

On the contrary, in other regions the stress field is substantially weakened
like in the situation shown in fig. 1.11 at B. There two cracks are parallel to each other and if the cracks are sufficiently long the stress between the two cracks will be screened, i.e. the external load will not be felt. From screening to attraction all scenarios are possible specially if more than two cracks are present.

The interesting question is, however, not just the stress distribution generated by the ensemble of microcracks but the growth and coalescence of these microcracks when the external load is increased, i.e. its dynamical evolution. Suppose that we increase the load from zero and reach the point when the first, i.e. the most stressed region fails, for instance that two close crack tips coalesce. Due to this coalescence the configuration of microcracks changes which for us means that the stress enhancement field is modified. Although we only consider linear elasticity the response of the system to the external load is now non-linear due to the changes in stress enhancement each time locally a region fails.

Non-linear problems of this kind are of course very difficult to handle and practically all that can be done are numerical simulations. Fortunately similar problems of non-linearities due to growth in essentially linear media are known from other areas, like viscous fingering in hydrodynamics or dielectric breakdown, dendritic growth or electrodeposition (see chapter 8). Recently physicists have been very interested in these problems ("growth models", "fractals") and a theoretical framework (scaling laws, etc.) has evolved. The description of this framework is one of the main tasks of this book.

While a precise calculation of the dynamical evolution of the ensemble of microcracks is technically difficult it is easy to give a qualitative description: The first regions that will fail, i.e. the most stressed ones, are determined by the initial position of the microcracks, i.e. their random location. But once in a region a crack has become larger and therefore the stress has been locally enhanced this region is more susceptible for further cracking. So regions that failed once tend to fail again. The effect of repeated failure will start to compete with the failure of the randomly placed, stressed regions that did not yet fail. When the repeated failure takes over one is roughly at the critical breaking load. The regions of multiple failure, i.e. the growing cracks also compete against each other because the larger cracks grow faster. Coalescence of cracks is the fastest mean of propagation. Finally one crack wins and rushes through the system coalescing with the cracks it encounters on its way. The time it takes for one crack to impose itself over the others strongly varies with the configurations of microcracks and this explains the large sample to sample variations encountered in the postfailure regime. The whole process can be visualized quite nicely if one tears apart a piece of swiss cheese!

This picture which in this simplicity is only valid for brittle fracture does of course not yet describe perfectly the reality because there exists a large variety of additional effects inside the materials. Let us just mention two, namely "crack arrest" and "healing". The "cohesive forces" which hold the material together and which the growing crack must overcome have a certain range and tend to pull the atoms together even if they are separated by a crack over several lattice units.
Consequently if a crack does not open up very much its stress enhancement can be substantially decreased even to the point of stopping the growth of a propagating crack. The most common reason for crack arrest is, however, when the crack tip encounters a heterogeneity in the system. If the two sides of a crack are pressed together the chemical bonds between the atoms can also be restored and a crack can heal. Chemical reactions or crystallization inside the crack opening are even more efficient mechanisms for healing. Both effects, crack arrest and healing, tend to stabilize the process of fracture.

1.4.3 The energy of fracture

The description we gave of brittle fracture is rather detailed but not easy to use for quantitative predictions. Alternative, more phenomenological approaches to fracture have been developed and used over the years. The most popular one concentrates on the energies involved in fracture and was first formulated in 1921 by Griffith.\(^{19}\) The idea is based on energy conservation. When a load is applied on a sample potential elastic energy is stored in the system. If a new crack is formed or an existing crack grows part of this elastic energy is released. On the other hand the formation or growth of a crack implies the creation of free surfaces, namely the crack surfaces which are energetically less favorable than the bulk because chemical bonds have to be broken up. So Griffith’s arguments states that a crack grows if and only if the release of potential energy is equal or larger than the surface energy that is required for the crack to grow.

Neither the elastic nor the surface energy are easily accessible in experiments or easy to calculate in general. But there are some cases in which calculations are feasible like the case of one single elliptical flaw of length \(a\) perpendicular to a uniaxial external displacement that is imposed on a plate (see fig. 1.10b). For mode I rupture one finds in the limit of large plates\(^{20}\) that the release of elastic energy \(dU\) caused by an elongation of the flaw by \(da\) is given by \(G = dU/da = \pi a \sigma^2 / E\) where \(\sigma\) is the externally applied stress and \(E\) is Young’s modulus. \(G\) is called the “energy release rate”. According to Griffith’s criterion the crack will grow if \(G\) equals the surface energy needed to create two surfaces of length \(da\) because a crack in a plate has two sides. In other words, the crack grows when \(G\) exceeds the critical energy release rate \(G_c\) which is given by \(G_c = 2 \gamma_s\) where \(\gamma_s\) is just the specific surface energy. Knowing the surface energy which is a material constant one knows therefore the critical stress for mode I cracking. The critical energy release rate \(G_c\) consequently plays a similar role as the toughness, i.e. depending on the rupture mode and on the material it determines when a single crack grows. It is in fact possible to write down a relation between the toughness and \(G_c\) (see chapter 5).

Up to this point it seems that we have just described in terms of energies what we did before in terms of forces. Energies are, however, global quantities while forces are locally measurable and instead of a local equilibrium of forces we used the principle of energy conservation. Conceptually one is therefore tempted to
generalize the Griffith criterion to arbitrary systems with many cracks and to use $G_c$ as a global, phenomenological parameter. The big drawback of this approach is that due to various effects happening in real materials energy conservation is usually not valid unless other contributions are taken into account. We already encountered the example of viscoelasticity which comes from energy dissipation within the material. Such a dissipation can be seen as an internal friction which can for instance heat up the system. In the energy balance dissipation makes up for an additional, \textit{a priori} unknown term that must be treated as another phenomenological constant and must be inserted by hand.

A particularly common loss of energy comes from the plastic regions around the crack tips since every plastic deformation requires work. Again a phenomenological term, the "plastic deformation energy" $\gamma_p$ can be introduced\cite{21} and the Griffith criterion for fracture is then modified to $G_c = \gamma_s + \gamma_p$. Unfortunately in many cases this is not just a small correction to the usual Griffith criterion because for most metals and polymers $\gamma_p$ is several orders of magnitude larger than $\gamma_s$. It seems nearly impossible to calculate $\gamma_p$ or $\gamma_s$ from first principles.

Another interesting problem related to the non-local nature of energy conservation is energy transport. The elastic energy freed by the growth of the crack is stored within a certain region and must be transported to the place where it is needed. There are several possible mechanisms that can assure the transport of elastic energy one being "phonons", i.e. waves inside the bulk like shear waves, but the most common being "Rayleigh waves"\cite{22} on the crack surface. Each mechanism occurs with a certain velocity which for phonons is for instance the sound velocity, i.e. $(1.5 - 12) \times 10^3 \text{m/s}$ and which for Rayleigh waves is about $(1.5 - 4) \times 10^3 \text{m/s}$. The existence of a finite speed for energy propagation introduces a time delay into the energy balance which is important if very fast processes are considered. Into the Griffith criterion these effects introduce an additional kinetic energy term.

If $G > G_c$, i.e. if more energy is available than the minimum needed for crack growth, the excess energy is often transformed into kinetic energy for the propagating crack. Consequently the faster the crack grows the more excess energy is available. We encountered this situation already when we discussed shocks. The existence of a finite velocity $v$ for energy transport imposes, however, a certain limitation on this picture.

If the crack propagates faster than $v$ the elastic energy which is released behind the crack tip and which can therefore never reach the tip accumulates on the sides of the crack. This accumulated energy tends to create more crack surface leading to side branching of the crack. These side branches are usually short and thus called "microbranching". If the velocity of the crack tip exceeds $\sqrt{2} v$ the crack tip can spontaneously bifurcate\cite{23} and two cracks simultaneously propagating with speed $v$ can be sustained as long as both are equally fast and one does not outrun the other ("macrobranching"). If enough energy is available each tip can again bifurcate and so one can finally have many simultaneously propagating tips. While microbranching usually follows grain boundaries or cleavage planes macrobranches
can be intergranular.

1.5 Materials

The effects that we have discussed up to now, like plasticity, corrosion, healing, etc., can in principle occur in all materials but their relative importance can vary over many orders of magnitude. The origin for these large variations can be found in the very different microscopic mechanisms that are involved in the mechanical response. The resulting difference in the behaviour of the materials makes up for the whole richness of fracture. In the following we will describe briefly the characteristics of various materials.

Roughly speaking one can distinguish three families of materials studied in fact by rather different communities: polymers, metals and what one may call "rock-alikes". The last category would include materials like concrete, glass, ceramics, heavy clay and tectonic plates. We will next give for each class some selected, certainly very incomplete information. In the next chapter experts will give more an insider view on some of the materials. The references will lead the reader to more details.

1.5.1 Glass

The fracture of glass has fascinated researchers for a long time. It was for glass that stress corrosion cracking\(^\text{[16]}\) was first diagnosticized and that the first theories for crack branching\(^\text{[23]}\) were conceived. Typically glass has a strength (breaking stress) under tension of 10–100 MPa but it is possible to prepare glasses which are more resistant to tensile fracture than steel (10 GPa). It is interesting to note that strength measured on supposedly identical samples shows already a typical scatter of 10 to 20\%. The slower the cracking process takes place the stronger is the temperature dependence on the breaking strength as shown in fig. 1.12. Until about 400 K the strength decreases with temperature due to "static fatigue", i.e. in this case due to corrosion induced by the slow hydrolysis of the silicates. At higher temperatures this process is counterbalanced by the desorption of water and the strength increases again. Finally the temperatures are so high that the glass fails by viscous flow. In the absence of an external stress the corrosion can increase the strength\(^\text{[2]}\) as much as 60\% because it blunts the cracks, i.e. it increases the radius of curvature at the crack tip.

Fracture of glass nearly always starts at the surface and under tensile stress. Therefore one can make glass more resistant to fracture by polishing its surface. Abrasion, i.e. surface wear, on the contrary, makes glass more fragile. The crack usually nucleates at some manmade imperfection. Its subsequent propagation leaves behind its fingerprint on the morphology of the crack surface. Around the initiation point one sees a flat region, the "mirror zone", normal to the direction of the tensile stress as shown in fig. 1.13a. The crossing of the rupture front with elastic waves can leave behind ripples in the mirror zone called "Waller lines". The mirror zone is delimited by the "mist zone" which consists of fine stirations looking
Figure 1.12 Breaking strength $\sigma_0$ of massive glass normalized by its value at 77 K as a function of temperature for two different failure times.

like microscopic blades oblique to the crack plane. The mist zone appears when the crack reaches speeds of about half the velocity of transverse elastic waves. It is the onset to a region of crack bifurcation, surface roughening and even shattering of the glass, called the "hackle zone", which occurs within a wedge-shaped chip (fig. 1.13b). Shape and size of the zone can be related to the stress distribution\textsuperscript{[24]} and the depth of the mirror zone seems to go roughly like the inverse squared tensile breaking strength. The speed of crack propagation can vary by about eight orders of magnitude.

1.5.2 Rocks

Geology is particularly interested in the fracture of rocks.\textsuperscript{[25]} Evidently the mechanical behaviour of rocks and minerals can be very varied. Let us just list some typical values for the stress intensity resistance in mode I measured in MPa $\sqrt{m}$: 0.01 to 0.3 for coal, 0.2 for calcite, 2.4 for natural quartz and about 3 for granite and sandstone. In fact, these values are strongly dependent on crack velocity, moisture, temperature, pressure and the geometry of the performed test as illustrated in the example of fig. 1.14. In addition, rocks can be very anisotropic. For example, the tensile strength of shale measured normal or perpendicular to the bedding may vary by a factor ten, the compressive strengths vary less.

The most common tests on rocks are performed under compression because on one hand this mimicks the enormous pressures to which rocks are submitted deep inside the earth's crust and on the other hand because of important techniques like hydraulic fracturing or deep explosions. Hydraulic fracturing in which fluid is injected into the rock under pressure is a way to improve the flow of oil and explosions are used in seismic measurements. The brittle to ductile transition was also discovered\textsuperscript{[8]} in compression experiments.
Figure 1.13 Morphology of a crack through glass; (a) top-view of the crack surface, (b) three-dimensional view of the wedge-shaped chip of hackle.

Compression experiments are often performed by applying a “confining pressure” $\sigma_1$ from all sides and putting in addition a “differential stress” $\sigma_3 - \sigma_1$ in the $z$-direction\textsuperscript{[11]} (see also fig. 1.4b where $\sigma_3 - \sigma_1$ is called $F/A$). For $\sigma_1 = 0$ igneous rocks typically break at $\sigma_3 = 100$–$200$ MPa while high porosity sediments break at about $\sigma_3 = 10$ MPa. If the confining pressure is increased the material becomes tougher. This can be seen for igneous rocks in fig. 1.15a which shows the increase of the critical differential stress as a function of $\sigma_1$.

The applied differential stress can also be a shear and in this case the confining pressure increases the strength of the sample even more than in the case of compression. Under shear the cracks have an inclination with respect to the $z$-axis and the angle $\phi$ of this inclination depends on the angle $\theta$ between the force $F$ of fig. 1.4b and the $z$-axis as illustrated in fig. 1.15b. Shear fracture under a confining pressure is encountered when a tectonic fault breaks up and starts moving which is the principal source for earthquakes. The sliding of the two faces of the crack surface on each other in the presence of pressure is called "slip". The asperities of the crack surfaces introduce a friction force during the slip motion. This friction decreases with the amount of sliding $\Delta L/L$, an effect called “slip-weakening", because the asperities are sheared off and crushed by the motion. The resulting debris found inside the crack is called “gouge" in the case of tectonic faults. Moving like an avalanche of sand ("cataclastic flow") gouge can act as a lubricifying fluid between the faces of the crack allowing for the creep of faults. Theoretical understanding of this process is still very poor. In fig. 1.16 we see slip-weakening measured on granite for various confining pressures.
Figure 1.14 Log-log plot of the stress intensity factor $K_I$ in mode I as a function of the crack speed for different quartz rocks at different temperatures measured either in water or in air (taken from ref. [25]).

Figure 1.15 Typical behaviour of igneous rocks. (a) Dependence of the differential fracture stress $\sigma_3 - \sigma_1$ at which the system breaks on the confining pressure $\sigma_1$ in compression experiments. (b) Dependence of the inclination angle $\phi$ of the crack on the angle $\theta$ that the applied force forms with the $x$-axis under shear.
5. Materials

![Graph showing slip-weakening](image)

Figure 1.16 Slip-weakening measured on initially intact granite for various confining pressures $\sigma_1$. We show the differential shear load $\sigma_3 - \sigma_1$ as a function of the amount of sliding $\Delta L/L$. The slip motion sets in once the maximum of the curve is reached.

Once the moving force behind the slip motion is gone because the stress that had built up in the tectonic plates has been relaxed by the motion of the fault, the crack surfaces stop moving, i.e. they "stick". Due to the drift of tectonic plates the stress slowly builds up again and the resulting "stick-slip" motion can be very complex. Already one-dimensional, deterministic models for stick-slip can show chaotic behaviour\textsuperscript{[26]} due to the decrease of the frictional force with the sliding velocity. A consequence of these simple models is not only the unpredictability of earthquakes but also a power-law distribution for the lengths and the amount of sliding $\Delta L/L$ of individual slips.\textsuperscript{[26]} Stick-slip motion is also encountered when two pieces of rubber are moved against each other under pressure. The stick-slip behaviour can be suppressed either by lowering the confining pressure or by increasing the temperature. If a fault sticks long enough it can heal under the enormous pressures that exist in the earth's crust due to a process that resembles the formation of sedimentary rocks. The recracking of healed cracks which is common in fault zones is a difficult theoretical problem.

Laboratory tests on rocks are not simple. Often rather big samples are needed in order to avoid too large sample to sample variations because the heterogeneities in rocks can reach the size of centimeters. Another typical problem encountered by geologists is that when a sample is removed from its in situ stress environment fractures can develop inside changing its behaviour in a test. Uniaxial
tensile strength is not easy to measure because in this case it is difficult to maintain the external load uniform.

Microscopically the typical defects that nucleate cracks in rocks have an opening but no shear displacement and only propagate in mode I. They can be "joints" that are formed during the cooling of igneous rocks, the drying of sediments or as tension gashes during faulting. They can also be about 1000 μm long and 1 μm wide penny-shaped microcracks within grain boundaries or intergranular cleavage planes.

These microscopic flaws propagate due to mechanical deformations but also due to thermal expansion, both common in the earth's crust. While propagating in mode I the surface of the cracks shows characteristic tilt and twist configurations due to the existence of shear fields. On larger scales mode I fracture in rocks also shows the mirror-mist-hackle scenario described above for glass. If a crack has grown long enough it can also propagate in mode II or mode III. In this case the surface morphology is less unique: In some rocks like sandstones one sees deformation bands, in other cases the crack is accompanied by plumes of microcracks called "feather fractures".

There is a delicate balance between the stress required to cause a mineral grain to cleave and the stress required to cause brittle grain boundary cracking. This balance can be upset by small changes in temperature, impurity content and rock texture. The fracture path can therefore be quite complex on large scales.

The presence of fluids, usually water, can have various effects. The high pressure of fluids enclosed in the pores of igneous rocks can produce strong local stress fields and the presence of such local constraints is known\cite{27} to change the elastic response of a system. They evidently also have an important influence on the brittle fracture behaviour. Healing is another effect that typically requires the presence of fluids. Hydrothermal processes including precipitation and solution under pressure can rapidly fill open microcracks with "cement".\cite{28}

Fracture of rocks is usually brittle. But we already saw that under pressure a transition to ductility can occur.\cite{29} Geologists concerned with deformation and failure of the earth's crust, consider pressures as high as 5 GPa when sedimentary rocks are plastic while igneous rocks behave relatively elastic. High temperatures also enhance the tendency to ductility. Two mechanisms can sustain plastic flow in rocks. On one hand one can have slip and twinning of crystal planes on each other giving rise to crystal plasticity. On the other hand one can have the microscopically brittle mechanism of cataclastic flow where the material slides on the debris of local fragmentation. Fragmentation will be the subject of the last chapter of this book.

1.5.3 Ceramics

Ceramics is a large class of man-made materials to which, in an extended sense, also belong glass and concrete. What crack resistance is concerned the spectrum is very wide for ceramics.\cite{29} There exist some heat insulators that can be crushed between the fingers while a mixture of titanium boride and boron carbide
can attain a compressive strength of 4 GPa; porcelain has 250–350 MPa. Tensile strengths are typically eight to ten times lower.

Usually ceramics are brittle and in this case, like in glass, cracks are generally initiated at surface flaws. Consequently the strength of brittle ceramics can be increased reducing the presence of these flaws for instance by flame-polishing or etching the surface. For a certain class of ceramics, called alumina (Al₂O₃), the strength can be varied through surface treatment between 0.5 GPa and 15 GPa.

We saw in a previous section that an increase in the strength can lead from brittle to ductile rupture. This is the origin of the "Joffe effect" of alkali halides,[30] they are brittle in air and ductile in water. The water polishes away surface imperfections of the sample by solution increasing the strength and rendering it ductile. When the sample is then dried small crystallites precipitate on its surface lowering the strength again and it rebrittles. Materials that behave in such a way are often called "semibrittle" because their yield strength and their fracture strength are close. Most ceramics are hard and are semibrittle only at high temperatures. Soft ceramics can be semibrittle at low temperatures. Ductility at low temperatures occurs for silver chloride and bromide but is generally very rare for ceramics. The dominant mechanism in ductile rupture is crystal plasticity.

A typical problem of material science is to increase the toughness of a given ceramics. We already discussed the effects of surface treatment. Another possibility is to change the internal structure of the material. In fig. 1.17 we see schematically how the breaking strength depends on the average grain diameter \( \bar{d} \) for various oxides. Two different behaviours are observed, the transition between the two being temperature dependent. At small grain sizes intergranular cracking
dominates and for large grains one has mainly intragranular effects.

The probably most efficient way to increase the toughness is mixing various components, i.e. to make "composites". Concrete is a well-known example. Spectacular are also the fracture strengths of more than 1 GPa for mixtures of ceramics and metals, called "cermets". The reinforcing of ceramics with fibers is another technique which not only increases the breaking strength but also changes the post-failure behaviour. This is illustrated in fig. 1.18 for glass reinforced with carbon fibers. The carbon fibers avoid catastrophic failure because they often do not break where the glass breaks but have to be pulled out of the glass by the opening crack. More details will be given about these problems in the sections on ceramics, composites and concrete of the next chapter and models for the rupture of carbon fibers will appear in chapter 6.

1.5.4 Polymers

In contrast to rock-alikes, polymers and metals are typically ductile and for this reason beyond the scope of most statistical models of this book with the exception of chapter 9. Their behaviour is, however, so rich that we will discuss some aspects here and two sections will be dedicated to them in the next chapter.

The rupture of polymers\cite{52} is usually and most easily studied under tension as the reader can experiment with a rubber band. Their failure behaviour can depend very much on temperature as shown schematically in fig. 1.19. At very low temperatures they fail brittle (a) and when the temperature is increased a transition to ductility takes place (b) with the usual necking instability occurring when the "ultimate stress" is reached. A neck can be initiated either because at some point the cross section of the sample is a little smaller or because a fluctuation in the material properties may cause a local reduction of the yield stress.
The higher the temperature the more the material can be stretched and the more the polymer chains become oriented in particular in the neck region. This orientation produces strain-hardening which stiffens the material and stabilizes the neck again. As a consequence the neck propagates along the length of the sample instead of strangling it, an effect called "cold-drawing" (c). In this situation elongations of 300–1000% can be achieved before rupture.

Finally at even higher temperatures the polymer undergoes a transition (glass transition) to rubber-like behaviour (d) in which the material can be elastically stretched over several times its size. Here rupture can be understood by visualizing the rubber as a network of crosslinked polymer chains similar to a random fabric. Small cracks nucleate at the weakest crosslinks until one dominating crack tears the fabric apart.

Since polymers are usually viscoplastic their rupture behaviour strongly depends on the "strain rate" $\dot{\varepsilon}$, i.e. the speed with which the elongation takes place. In fig. 1.20 we see how the tensile strength $\sigma_e$ and the "ultimate strain" $\varepsilon_u$, i.e. the elongation at which necking sets in, depend on $\dot{\varepsilon}$ for a certain type of rubber.

An interesting phenomenon that often precures polymer fracture is the appearance of "craze". Craze is a dense network of fine planar defects often starting at the surface of the sample. These defects lie orthogonal to the direction of the tension. They contain about 50% free space and the rest are oriented polymer chains cutting through the planes of these defects. The formation of craze can be
Figure 1.20 Variation of the tensile strength $\sigma_c$ (full line) and the ultimate strain $\varepsilon_u$ (dotted line) of a rubber with strain rate $\dot{\varepsilon}$. The strain rate is in fact multiplied with a temperature-dependent factor $\alpha_T$ such that the data points fall on the shown curves for all temperatures $T$ (see ref. [32]).

explained by what is called “cavitation”, namely a spontaneous appearance of voids that happens in some plastic media under tensile stress.

Under cyclic fatigue heat is generated in polymers due to the relative motion of adjacent chains. Since polymers are bad heat conductors the temperature of the sample rises. The value $T$ of this temperature depends on the frequency as shown in fig. 1.21 for polyethylene. The temperature rise can lead to thermal softening failures.

1.5.5 Metals

Because of the technological importance of metals their fracture behaviour has been studied most intensely also at rather microscopic levels. The clue to the understanding of metal failure is the motion of dislocations.

“Slip”, i.e. the translation by a multiple of the lattice constant of one crystal plane on the other is equivalent to the motion of an edge dislocation along the plane. Since real metal grains are usually full of interpenetrating dislocations many complex relative motions are possible inside a grain, giving rise to the mechanism that underlies plasticity. Dislocations can be locked by present impurities in the form of a slip band, called “Lüders band”. If the stress is increased the dislocations or a Lüders band can unlock and sweep through the grain generating plastic flow. Under sufficiently high stress also “twin”, i.e. relative motions of crystal planes by fractions of units of the lattice constant can occur.
The collective motion of dislocations produced by the deformation of the metal grains can generate interesting spatial patterns like regular networks of dislocation bands or a self-similar decomposition of the Lüders bands into fine subbands. A continuum description of the motion of the density of dislocations in reaction-diffusion equations can explain some of these patterns.\textsuperscript{[36]}

Due to the mechanisms that we described one of the keys to increase the strength of metals is controlling the impurities. Strong steel is obtained from iron by adding carbon and certain metals. The tensile strength of a piano wire can reach 300 GPa which is, however, still one order of magnitude smaller than that of the strongest silica fibers. Impurities can also substantially lower the strength of metals as happens in the hydrogen embrittlement mentioned in section 3 or in the “blue brittleness” produced by the presence of nitrogen in carbon steel. Consequently the strength of alloys of iron and nitrogen can be as low as 0.5 GPa.

The list of materials for which fracture has been studied is by far not exhausted. The fibrous and cellular structure of biological substances\textsuperscript{[13]} like wood\textsuperscript{[87]} and bone\textsuperscript{[2]} can have rather exceptional breaking properties. The tearing of paper and textiles is of interest to certain industries and the rupture of ice\textsuperscript{[2]} has been investigated by geoscientists.
1.6 Outlook

We have given a very brief and therefore superficial overview of the traditional knowledge about fracture. The aim of this chapter was to give the non-specialist reader a feeling for the richness of the field and to guide him through the current terminology.

With the exception of the next two chapters the rest of this book is not intended to work out the details of what was said here but to present less traditional approaches. In fact, most of the material presented in this chapter is rather well-established as can be seen from the age of many of the references. Nearly all the subjects that have been touched here are elaborated pedagogically and in detail in many textbooks, some of them having been listed as references.

I thank D. François and D. Stauffer for critical reading of the manuscript.

References

17. see e.g. M.G. Fontana in Stress Corrosion Cracking in Aircraft Structural Material, (NATO, AGARD Conf. Proc. Ser. No. 18, 1967)
19. A.A. Griffith, Phil. Trans. Roy. Soc. of London 221, 163 (1921)
23. E.H. Yoffe, Phil. Mag. 42, 739 (1951)
31. W. Weibull, Kgl. Svenska Vetenskapsakad Handl. 151, (Stockholm, 1939)