FINITE SIZE SCALING APPROACH TO A METAMAGNETIC MODEL IN TWO DIMENSIONS

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We investigate the tricritical properties of a metamagnetic model, namely the next-nearest neighbor Ising antiferromagnet, in two dimensions. We calculate the transfer matrix on finite strips and use finite size scaling to obtain the critical line. The tricritical point and its exponents are obtained by two different methods. In the case of strong intersublattice coupling no evidence for tricritical behavior is found.

Metamagnetic models are models in which two sublattices with competing intra- and inter-sublattice couplings are considered in a homogeneous field. They have been of interest for a long time because they show tricritical behavior [1–5] and have a variety of experimental realizations [6]. In this paper we will specifically consider the two dimensional next-nearest neighbor (nnn) Ising antiferromagnet on a square lattice defined by the Hamiltonian

\[ \mathcal{H} = J \sum_{\text{nn}} \sigma_i \sigma_j - J' \sum_{\text{nnn}} \sigma_i \sigma_k + H \sum_i \sigma_i, \]

(1)

where \( \sigma_i = \pm 1 \) are Ising variables and \( J, J' > 0 \). The first sum describes the antiferromagnetic intrasublattice coupling between nearest neighbors and the second sum is a ferromagnetic intrasublattice coupling between next-nearest neighbors. In the temperature–field \( (T-H) \) plane one expects a transition line which is of first order for small \( T \) and Ising-like for small \( H \) (fig. 1). The point at which the transition changes from first to second order is usually expected to be a tricritical point. It should, however, be noted that mean field theory [4] only predicts a tricritical point for \( R = J'/J > 3/5 \). For \( R < 3/5 \) in the mean field approximation two Ising-like critical endpoints appear. But in the range of \( R \) which has been considered numerically up to now \( (R > 1/8 \; [7]) \) only evidence for tricritical behavior and no decomposition into two critical endpoints has been found. The most accurate numerical determination of the tricritical point and its exponents was performed for \( R = 1/2 \) with an extended Monte Carlo renormalization [5]. For the sake of comparison we will therefore also consider \( R = 1/2 \) and in order to check the mean field prediction for small \( R \) we also look at \( R = 1/50 \).

We investigate the described model by means of a phenomenological renormalization [8], i.e. we calculate exactly the correlation length of the model on a strip of infinite length and width \( n \) and extrapolate to infinite \( n \) with the help of finite size scaling. The exact calculation is performed with a transfer matrix which transfers from one column of the strip to the next. The transfer matrix itself is calculated through a factorization in \( n \) operations. We impose periodic boundary conditions on the strip so that the \( (n+1) \) th site of a column is identical to the first site of the same column. Only even values for \( n \) are considered in order to have two independent sublattices. The correlation length is calculated as [8]

\[ \xi = \log^{-1} (\lambda_1 / \lambda_2), \]

(2)

where \( \lambda_1 \) is the largest, \( \lambda_2 \) the second largest eigenvalue of the transfer matrix. As the eigenvector of \( \lambda_2 \) is only translation invariant with respect to translations within a sublattice the size of the transfer matrix cannot be reduced by symmetry as much as for the Ising model and the largest strip width we calculate has a width \( n = 10 \).

First we make the usual assumption of the phenomenological renormalization that
$n^{-1} \xi_n(H_c(T)) = m^{-1} \xi_m(H_c(T))$ (3)

for the correlation length $\xi_n$ of the strip of width $n$ determines the critical field $H_c(T)/J$ as a function of $T/J$. The result for $m = n - 2$ is shown in fig. 1 for different $n$ and $R$. The typical region at low temperatures where the critical field is nearly constant is 4 and where the first order transition occurs becomes very small for $R = 1/50$ and the phase diagram for $R = 1/50$ becomes very similar to that of the pure antiferromagnet [9].

To determine the critical point $(T_{cr}, H_{cr})$ an additional equation to eq. (3) is required [10]. One possibility is to consider three different strip widths,

$n^{-1} \xi_n(T_{cr}, H_{cr}) = m^{-1} \xi_m(T_{cr}, H_{cr}) = l^{-1} \xi_l(T_{cr}, H_{cr})$, (4)

which we call method A. Another possibility is to consider an equation of the type of eq. (3) for the second correlation length,

$\xi = \log^{-1}(\lambda_1/\lambda_3)$, (5)

where $\lambda_3$ is the third largest eigenvalue; we call this method B. In fig. 2 we show the values of $T_{cr}/J$ and $H_{cr}/J$ calculated with both methods for different strip widths. We extrapolate $T_{cr}/J = 1.210 \pm 0.008$ and $H_{cr}/J = 3.965 \pm 0.002$ which is nicely within the error bars of ref. [5].

The tricritical exponents $y_1$ and $y_2$ are determined in the same way as in ref. [10], i.e., using the derivatives of $\xi_n$ with respect to $H$ and $T$ for different strip widths $n$ eliminating the constants $a_i$ and $b_i$ of the scaling fields.

![Graph showing temperature-field phase diagram with critical points and tricritical exponents determined.](image-url)
\[ u_2 = a_2(H - H_1) + b_2(T - T_1) + \ldots, \]
\[ u_1 = a_1(H - H_1) + b_1(T - T_1) + \ldots, \]

in the scaling relation
\[ \xi_n(H, T) \sim nF(n^{\nu_1}u_1, n^{\nu_2}u_2), \]

where \( F \) is a scaling function. In fig. 3 we show our values for \( \nu_1 \) and \( \nu_2 \) for \( R = 1/2 \) calculated with method B and extrapolate
\[ \nu_1 = 1.798 \pm 0.004, \quad \nu_2 = 0.805 \pm 0.015. \]

These values agree well with previous numerical work [5] and are clearly more precise. \( \nu_2 \) also agrees well with the exact value \( \nu_2 = 4/5 \) obtained for another model at the tricritical point [11] and which might be in the same universality class as the model considered here. The convergence in the case of method A is much slower than for method B.

Making the same calculation for \( R = 1/50 \) one obtains using method B one simultaneous fixed point at \( T_1/J = 0.1660 \pm 0.0016 \) and \( H_1/J = 3.941 \pm 0.003 \) which has only one relevant exponent \( \nu_1 = 1.005 \pm 0.010 \). This point is clearly Ising-like and not tricritical. For stripwidths \( n = 2, m = 4 \) one even finds a second Ising-like point at \( T_2/J = 0.275 \) and \( H_2/J = 3.92 \) but this point disappears for larger widths and is thus an effect of the small width. The point \( (T_1, H_1) \) could also in principle disappear at large widths but this seems highly improbable. We have calculated up to width 8. In the region \( 0.01 < T/J < 0.5 \) no other simultaneous fixed points exist for these widths. If a tricritical point exists in the region \( T/J < 0.01 \), which we cannot access for numerical reasons, the point \( (T_1, H_1) \) might be a fixed point of the second order line. If, however, no other simultaneous fixed points exist, which is probable \( (T_1, H_1) \) is the “tricritical” region and this would suggest that the mean-field picture is correct for \( R \) as small as 1/50.

After completion of this work we learned about a similar work performed by Rikvold et al. [12] which agrees in this conclusions with our results at \( R = 1/2 \).

In summary, we showed how to calculate tricritical exponents with the phenomenological renormalization and that this method is more accurate than other numerical methods. For the nnn Ising antiferromagnet we found some unexpected evidence that the mean field picture might be correct in two dimensions for small ratios \( R \), i.e. for strong intersublattice coupling.

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References