Effect of the Impact Energy on the Structure and Dynamics of 1D Piles

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Abstract

The influence of the impact energy on the structure and avalanche dynamics of a slowly driven 1D pile of spheres was experimentally studied. Dropping the beads from higher positions produces piles of higher packing density, smoothes their surfaces and modifies the avalanche size distributions. This results in effectively height-dependent critical exponents. Nevertheless, the SOC behavior, defined as the scaling of the avalanche size distribution for different lengths of the system, shows to be robust against variations in the dropping height.

1-Introduction

The sand pile scenario of “Self Organized Criticality” (SOC) [1, 2] is the following: When grains of sand are added onto a horizontal surface, a conical pile grows organizing itself through avalanches to attain the slope given by the angle of repose. If the addition of grains is slow enough a critical state is reached in which each grain can provoke an avalanche of any size and duration. The distribution of avalanche sizes and durations will follow a power law without the need of any external fine tuning. One observes finite-size scaling in both distributions and a "$1/f$" power spectrum. Self-organization and criticality are the basic pillars of Self Organized Criticality (SOC) [1, 2]. Several other phenomena in nature seem to show analogous behavior. This has provoked a substantial research activity[3, 4, 5], but the apparent simplicity of the sand pile has established this system as a paradigm for SOC behavior.

In spite of the large amount of theoretical work published on the subject, only few experiments have been carried out to check the SOC properties in real piles. These studies have revealed the need to fix some properties of the system, such as the grain type, the size of the pile and the type of base in order to obtain SOC behavior. Held and co-workers [6] noted that for large enough pile sizes, periodic big avalanches dominate the dynamics, in contradiction to the original SOC scheme. However, they claimed that SOC behavior was robust provided the piles were small enough. This was later confirmed in several experiments [7, 8, 9]. Frette et al. [10] and Christensen et al. [11] studied the avalanche dynamics in quasi-1D rice piles for differently shaped grains, and found that SOC behavior is attained also for large piles in case the inter-grain friction is strong enough to neglect the inertial effects which can be realized for grains with relatively high aspect ratio. In our previous work [12] we studied the influence of the base type on the avalanche dynamics of slowly driven 1D piles of beads, by adding spheres from a height of $10{\pm}2\,cm$ above the apex of the pile. We concluded that a base made of randomly spaced glued beads, generates a bulk disordered pile and a rough profile and exhibits SOC behaviour, i.e. produces avalanches which could be described by the critical exponents predicted by Malthe-Sørensen [13]. Bases which induce less disorder in the piles suppressed the SOC behavior.

In this work we concentrate on the study of the influence of the driving energy on the SOC behavior of 1D piles of beads. Held and co-workers [6] already noted slight variations in the avalanche distributions when adding grains at different heights to 2D sand piles. We use the randomly spaced base that produced SOC behavior in our previous paper [12], and vary the height from which beads are added to the pile. As the driving energy increases, the bulk of the pile becomes denser near the impact area, while the pile profile flattens and fluctuates less. This is accompanied by a decrease in the number of big avalanches and an increase in the number of small avalanches. Nevertheless, the quality of finite-size scaling is good for all heights, suggesting a robust SOC behavior.

2-Experimental Procedure

The experimental setup is the same as used in our previous work [12]. It consists of an acrylic strip sandwiched between two parallel vertical glass plates $5{\pm}0.2\,mm$ apart from each other so that a horizontal surface of $5 \times L\,mm^2$ (where L varied from 8 to 24 cm) was available for the formation of a quasi-1D pile of $4 \pm 0.005\,mm$ diameter steel beads. The base consists of a row of the same type of beads glued to the surface with random spacings (1, 2 or $3mm$) between beads. The beads were dropped one by one from a height h above the base. Both sides of the base were open, so the beads were able to fall off the system.

The falling events were detected by measuring the weight variations of the pile using a digital scale. The whole setup was computer controlled in such a way that a bead was added only after all motion coming from the previous impact had stopped. The measuring software identified an avalanche of size n when n beads fell off the pile after dropping one bead on the apex of the pile. A typical experiment included more than 30000 dropping events, with an average total duration of 80 hours. For the avalanche statistics, the events before the "steady" average pile size was attained, and the avalanches of size zero (no beads falling off the pile after a dropping event) were eliminated. The system failed to give a correct avalanche size (i.e., it one bead gave in excess or in defect) less than once in 500 dropping events. We obtained images of the pile every 500 dropping events by means of a digital camera.

We report results for three different heights: $h_1 = 105 \pm 1\,mm$, $h_2 = 190 \pm 1\,mm$ and $h_3 = 270 \pm 1\,mm$. All images are obtained for an $240mm$ base length as shown in Fig. 1. At least two runs of each height and base length were performed to assure the reproducibility of the results.

3-Results and Discussion

Fig. 1 displays the structure of a representative pile for $h = h_3$ with a $240mm$ base length. The heavy dots at the base represent the centers of beads glued to the acrylic strip. We construct a triangulation connecting the centers of touching beads to underline the structural features of the pile. The *profile* is given by the thick line that links consecutive beads on the pile's surface. The shadowed area has a width equal to 10% of the length of the base, and divides the pile in three zones: left, center (shadowed) and right.

3.1-Bulk structure

To quantitatively analyze the bulk structure of the piles we define for the triangulation shown in fig. 1 the angle ϕ between two consecutive segments around a given bead. Then we measure the histogram of ϕ, i.e. the number of angles having a certain value (not taking into account the beads along the profile or those at the base) and normalize it by the total number of angles. Finally the average and the standard deviation are calculated for all the 59 pile images computed for each height.

Fig. 2 shows the result of the above described analysis. One sees a maximum of the number of angles around $\phi = 60^o$ which increases with h. Since $\phi = 60^o$ corresponds to a regular triangular lattice this suggests more structural order as h increases. The analysis of the "*active zone*" of our piles shows that almost all the beads change their positions during the experiment except some very close to the base as expected in a SOC scenario [12]. The inset in Fig. 2 shows that also the variations in the structure are largest around $\phi = 60^o$.

The histogram of ϕ was calculated independently for the three different areas of the piles defined above (shown in Fig. 1). In each case, we fitted a Gaussian to the 60^o peak, and calculated its area. Fig. 3 shows the results, which are normalized to the analogous area corresponding to the regular triangular lattice, i.e. the most compact case. These data are obtained using a base of closely glued beads (called *"gap0"* in our previous work [12]). Fig. 3 shows that for sufficient impact energy the histograms are higher in the center of the piles (i.e., near the impact region) which means that *"order"* increases when the impact is stronger. It has been evidenced that vibrations provoke denser packings in 1D piles [14]. In our case, the dropping beads produce sufficient vibrations inside the pile, that spheres are locally rearranged to form structures of higher packing density. The small differences observed between the left and right zones are due to the details of the "quenched disorder" of the base

[12].

3.2-Profiles

The profiles averaged over all computed images for each height are displayed in Fig. 4. The increase of the absolute value of the slope near the ends of the base is due to the fact that there are less layers of grains, in agreement with Pouliquen's observations [15]. On the top the profiles become flatter, the more the higher the impact energy. Following the work of Alonso et al. [16] one can model this flattening effect by the loss of energy e of the down-jumping grains until they are finally stopped within the local minima U of the profile and become part of the pile. In its continuum form this model gives the differential equations[17]

$$\delta \, \frac{de}{dx} = (r-1)e(x) - r\gamma(x) \qquad (1)$$

$$\delta \, \frac{d\gamma}{dx} = \Gamma\big(U - e(x)\big) \; , \qquad (2)$$

where x is the spatial coordinate, δ the diameter of the bead, r the restitution coefficient, γ the slope $\frac{dy}{dx}$ of the pile multiplied by the energy unit $mg\delta$ (m being the mass of the beads) and Γ denotes a relaxation-parameter with $0 < \Gamma < 1$. The above linear equations can be coupled in one equation of second order. The energy of the beads gives an exponential decay starting with $e(0) = mgh_0$ and going to U for $x \to \infty$. For the equation of γ we choose the boundary conditions $\delta \, \frac{d}{dx}\gamma(0) = \Gamma(U - e(0))$ and $\gamma(L) = mg\delta \, \tan\theta_L$, one finally obtain for the profile height:

$$y(x) = y_L - \tan\theta_L \, (x - L) + \frac{G(x)}{mg\delta}, \qquad (3)$$

where

$$G(x) = \tfrac{C_1}{\lambda_1}\left(e^{\lambda_1 x} - e^{\lambda_1 L}\right) + \tfrac{C_2}{\lambda_2}\left(e^{\lambda_2 x} - e^{\lambda_2 L}\right)$$

$2\delta\lambda_{1,2} = r - 1 \pm \sqrt{(1-r)^2 + 4r\Gamma}$, and y_L and θ_L being respectively the height and angle of the profile at $x = L$. The threshold $U = \frac{\gamma(L)\; r}{1-r}$ coincides with the value of the energy of the beads at the ends of the base, guaranteeing the maintenance of the angle of repose. Inserting the physical parameters for δ, θ_L, y_L and mg one obtains for the three values of h_0 the solid lines shown in Fig. 4 using $r = 0.89$ and $\Gamma = 0.001$. We see that the agreement with the measured profiles is reasonable, which suggests that the shape of the flattening on the top is essentially a consequence of dissipation.

The profile activity was analyzed by means of the standard deviation of the profiles of all computed images (Fig. 5). The results are consistent with our results for the bulk structure: the variations in the profile are a little smaller near the impact area, where the bulk has a higher degree of order, and as the driving energy increases, the fluctuations in the profiles are smaller. The right area of Fig. 5a ($h = h_1$) shows more fluctuations, also in agreement with the disorder indicated by the smallest peak area in Fig. 3. These facts indicate that the structural disorder provokes fluctuations in the piles's profiles. One also observes in Fig. 5 a steep decrease of the fluctuations at both edges of the system which are clearly boundary effects.

3.3-Avalanche dynamics

Avalanches in sandpiles are caused by an accumulation of grains leading to a slope that exceeds the angle of repose. When this occurs, the addition of a single grain can trigger the liberation of potential energy in form of an avalanche. Consequently, the pile profile evolves in such a way that its average slope decreases. The avalanche mechanism is also related to the roughness of the profile shown in Fig. 5.

In Fig. 6 we see the avalanche size distribution for three different dropping heights. An asymptotic power law regime cannot be observed very well since the systems are small. The curves for the three heights are similar except that they become steeper with increasing height. This means that for larger impact energies the avalanches become typically smaller.

One possibility to explain this observation is to correlate the avalanche statistics to the pile's structure. For $h = h_1$ the energy of the dropping beads is relatively small. As seen in the previous sections this allows the formation of more disordered bulk and profile structures on the top of the pile and so a larger accumulation of beads in the profiles. As h increases, the impact of the dropping beads is stronger which provokes a denser packing in the pile's structure. Due to the smoother profiles and the existence of more ordered structures, the average bead accumulations in the profiles decreases. This implies an increment in the number of small avalanches and a decrement in the number of the big ones as h increases as shown in Fig. 6.

Another explanation could be that the larger impact energies produce more vibrations in the pile therefore reducing the difference between the static and the dynamic angle of repose and so generating more but smaller avalanches. Finally one could also explain the reduction in the number of large avalanches for large impact heights by the fact that in that case the impacting beads can perform large jumps towards the edge of the system and therefore reduce the effect of accumulation of grains at the top.

Figs. 7a, c and e display the avalanche distributions for three different base lengths for each height. They are normalized by the total number of nonzero avalanches. In spite of the larger number of big avalanches when $h = h_1$, the three plots have similar features. When we apply to the previous distributions the finite-size scaling ansatz

$$P(s, L) = L^{-\beta} f(s/L^{\nu}) \qquad (4)$$

we obtain the data collapses shown in Figs. 7b, d and f. The normalization condition $\langle s \rangle = \int P(s)ds = 1$ [12] implies $\beta = \nu$. Due to the differences in the slopes of the avalanche distributions, the critical exponents of the scaling relations have to be different for each height. The best collapses were displayed with $\beta = \nu = 1.5$ for h_1, $\beta = \nu = 1.3$ for h_2 and $\beta = \nu = 1.2$ for h_3. The values $\beta = \nu = 1.35$ obtained in [12] are in agreement with our results because there, the beads were dropped from a height $10 \pm 2cm$ above the apex of the pile, which corresponds approximately to h equal $150mm$ above the beads of the base in this experiment. In the present study a cursory inspection shows the similarity and good quality in the collapse of the avalanche distributions for the three different heights. This shows that variations in the driving energy do not affect the SOC behavior of our piles, but change its critical exponent.

4-Conclusions

We showed experimentally that the scaling of the avalanche size distribution in slowly driven 1D piles of beads is robust with respect to the driving energy, although it influences the value of the corresponding critical exponents. These facts can be correlated to our structural observations. Our findings, in addition to previous reports, establish that the type of grains and the nature of the base are the main ingredients that can actually destroy SOC in real 1D piles, while the driving energy -within certain limits- just modifies its details.

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Figure Captions

Fig. 1. Schematic representation of a typical pile for $h = h_3$ with a $240mm$ base length. We see the triangulation obtained by connecting the centers of the beads underlining the structural feature of the pile. The shadowed area has a width equal to 10% of the length of the base, and divides the pile in three zones: left, center (shadowed) and right.

Fig. 2. Average and standard deviation (inset) of the histogram of ϕ averaged over all computed images for each dropping height (normalized by the total number of angles).

Fig. 3. Areas of the peaks of the histogram at different zones of the piles for each dropping height h_0.

Fig. 4. Profiles averaged over all images for each dropping height. The solid lines fit the data according to eq. (3).

Fig. 5. Standard deviation of the profiles over all computed images for each dropping height.

Fig. 6. Avalanche size distributions (normalized by the total number of dropping events) for each dropping height, corresponding to a $240mm$ base length.

Fig. 7. Avalanche size distribution (normalized by the total number of nonzero avalanches) for (a) $h_1 = 105mm$, (c) $h_2 = 190mm$ and (e) $h_3 = 270mm$. The scaling based on the ansatz $P(s, L) = L^{-\beta} f(s/L^{\nu})$ is shown in (b) h_1 ; $\beta = \nu = 1.5$, (d) h_2 ; $\beta = \nu = 1.3$ and (f) $= h_3$; $\beta = \nu = 1.2$.