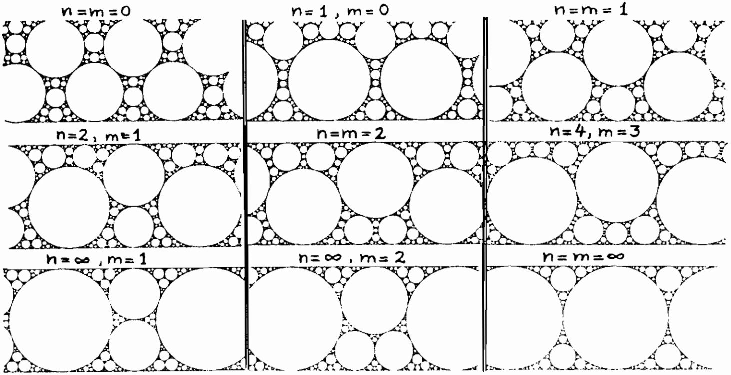
Fractal bearings

The problem of tiling the space between three touching circles with an infinite number of smaller and smaller circles is known as "Apollonian packing" and dates back to 200 BC. The problem has now been extended by Hans Herrmann and colleagues at CEN Saclay, France, to the case of rotating disks in a strip of finite width and infinite length (*Phys. Rev. Lett.* (1990) **65** 3223). Surprisingly this seemingly abstract geometric problem can be related to problems in plate tectonics and turbulence and, not surprisingly, fractals.

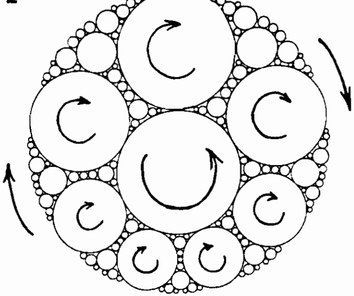
The research was prompted by the observation that the Earth's crustal plates can move over each other without causing earthquakes or producing heat through friction. This could be explained schematically if the material between the plates, the "gouge", was organised in rolling disks such that any rotating disk is only in contact with disks rotating in the opposite direction. Since these "bearings" have rolling friction, but not gliding friction, the lack of measurable heat production and seismic activity can be explained. Such a tiling would have to be constructed iteratively and would therefore be self-similar. Obviously this is a simplified representation of the dynamics involved but observations have found self-similar motion within the gouge.

The tiling is achieved by successively applying three different Möbius transformations in the complex z-plane (of the form $z' = (az + b) / (cz + d)$ where a, b, c and d are constants) to a pair of touching circles, thereby progressively filling the wedge between them. For example, in tilings where it is possible to connect loops of four rotating disks within the pattern (loops must comprise an even number of disks to ensure any disk is only in contact with disks rotating in the opposite direction) there are two families of tiling corresponding to different "initial conditions" and Möbius transformations. The tilings within each family can be characterised by two integers, m and n, running from zero to infinity, see figure 1. A similar hierarchy is expected in tilings of six-fold loops and beyond. In figure 2 a four-fold tiling with $n = m = 4$ has been conformally mapped from strip to circular geometry.

1



2



Herrmann and colleagues find that the fractal dimension of such space-fillings are consistent with the Kolmogoroff scaling of the energy of fully developed turbulence, suggesting a "geometrical interpretation of turbulence as a picture of energy transfer to smaller and smaller eddies in the inertial regime".

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